The Tensor Algebra Compiler

$$a = Bc$$
$$A = B \quad a = B^T c$$
$$a = Bc + a \quad a = Bc + b \quad a = b + c$$
$$A_{ij} = \sum_k B_{ijk} * c_k \quad a = B^T c + d \quad a = \alpha Bc + \beta a$$
$$a = BCd \quad A = B + C \quad A = B \odot C \quad A = BC \quad A = 0 \quad A = B^T \quad A = (B + C)d$$
$$A = \alpha B \quad A = \alpha A + \beta B \quad A = \alpha B + A \quad A = A + \alpha I \quad A = B + C + D$$
$$A = (B + C)(d + e) \quad A_{ik} = \sum_j B_{ijk} * c_j \quad A_{ij} = \sum_k B_{ijk} * c_k \quad A_{ilk} = \sum_j B_{ijk} * C_{lj} \quad A_{il} = \sum_{j,k} B_{ijk} * C_{jl} * D_{kl}$$
$$A_{jl} = \sum_{i,k} B_{ijk} * C_{il} * D_{kl} \quad A_{jl} = \sum_{i,k} B_{ijk} * C_{il} * D_{kl} \quad A_{ik} = \sum_i B_{ijk} * c_j \quad A_{jl} = \sum_{i,k} B_{ijk} * C_{il} * D_{kl} \quad A_{ijl} = \sum_k B_{ijk} * C_{lk}$$
$$A_{ij} = B_{ij}(C_{ik}D_{kj}) \quad A_{ij} = B_{ij} \sum_k (C_{ik}D_{kj}) \quad A_{ij} = \sum_k B_{ijk}C_{ijk} + D_{ij} \quad A_{ij} = \sum_k C_{ijk}D_{ijk} \quad A_{ij} = \sum_{l,k} B_{lik} * C_{lj} * D_{kj} \quad A_{ij} = \sum_{k,l} B_{ikjl} * C_{kl}$$

Fredrik Kjolstad, Shoaib Kamil, Stephen Chou, David Lugato, and Saman Amarasinghe

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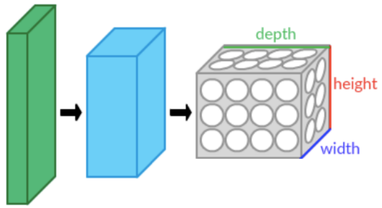
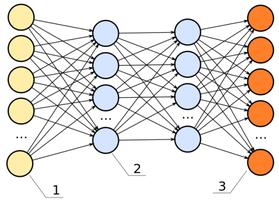


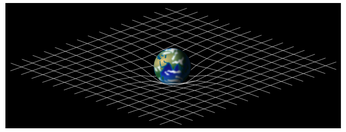
Convolutional Layer

Images

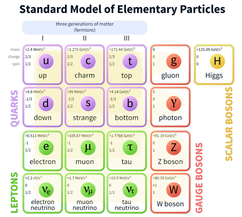


Connected Layer

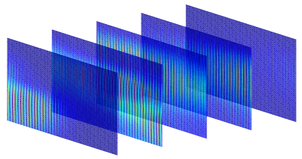
Science and Engineering



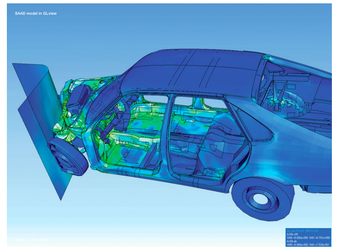
General Relativity



QCD



Fluorescence spectroscopy



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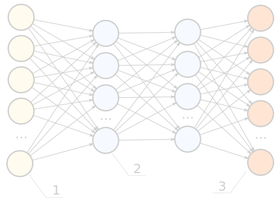

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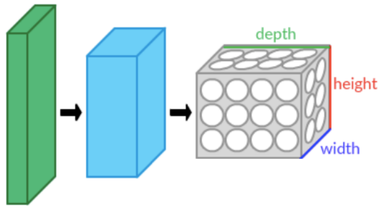
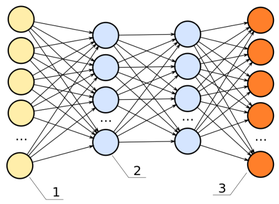


Convolutional Layer

Images



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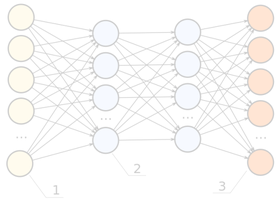

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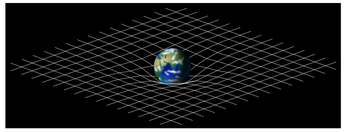
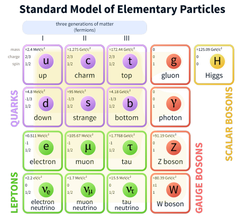
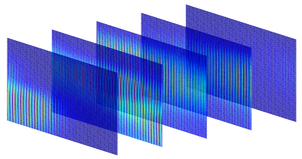


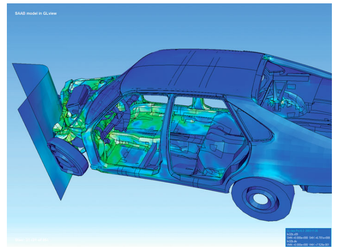
General Relativity



QCD



Fluorescence spectroscopy



Finite Element Method

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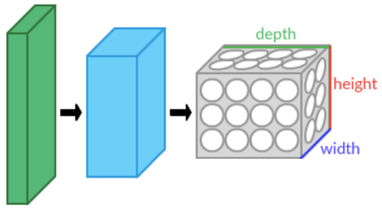
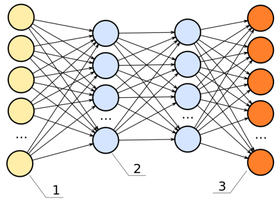


Convolutional Layer

Images



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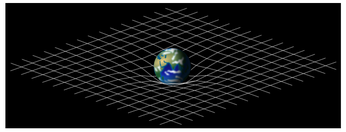

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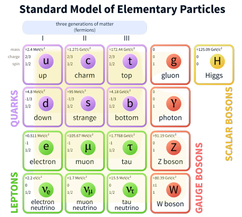

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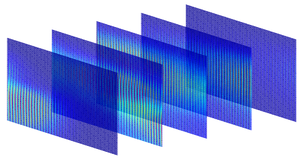
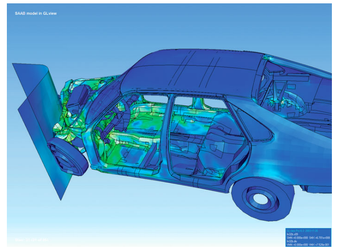


Fluorescence spectroscopy



Finite Element Method

Scalars have 0 dimensions
Vectors have 1 dimension
Matrices have 2 dimensions
An n-tensor has n dimensions

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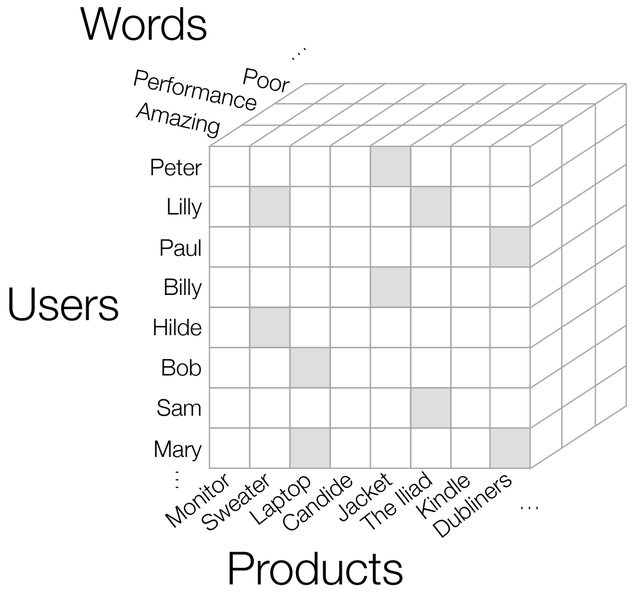

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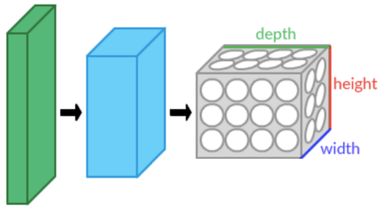
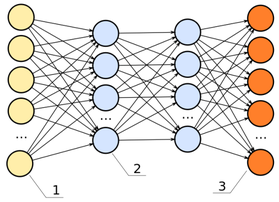


Convolutional Layer

Images



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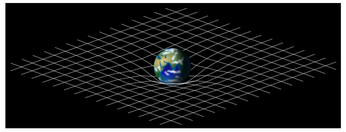
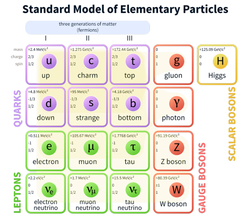
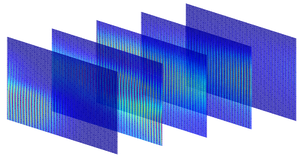


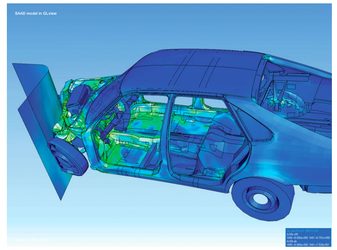
General Relativity



QCD



Fluorescence spectroscopy



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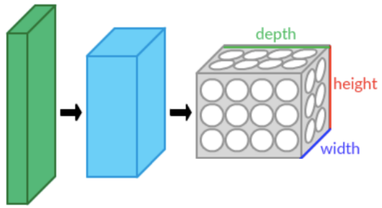
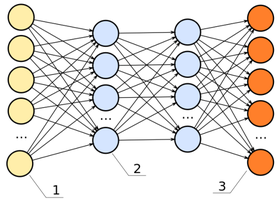


Convolutional Layer

Images



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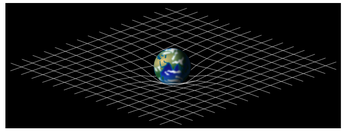
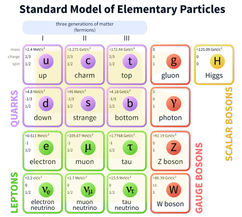


General Relativity



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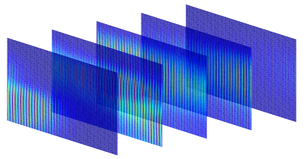
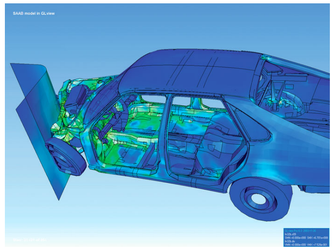


Fluorescence spectroscopy



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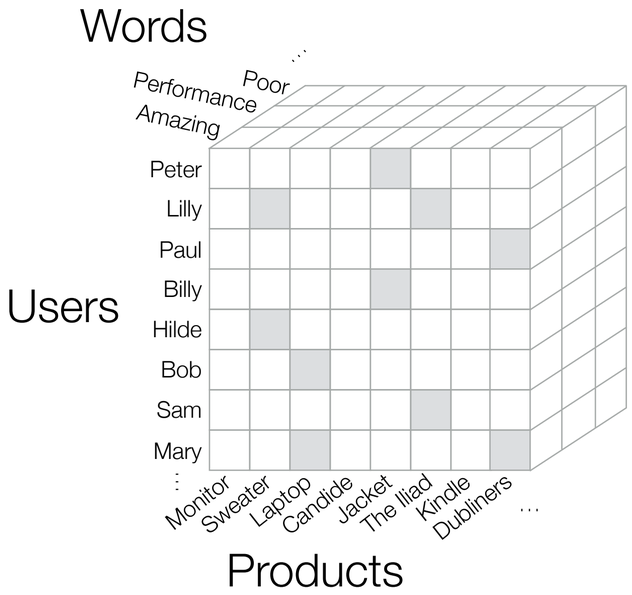

Extremely sparse
Dense storage: 107 Exabytes
Sparse storage: 13 Gigabytes

We need a Tensor Algebra Compiler

Number of Variants:

We need a Tensor Algebra Compiler

$$a = Bc$$

Number of Variants: 1

We need a Tensor Algebra Compiler

$a = Bc$ Eigen

Number of Variants: 1

We need a Tensor Algebra Compiler

$a = Bc$ Eigen

$a = Bc + a$ CSparse

Number of Variants: 2

We need a Tensor Algebra Compiler

$a = Bc$ Eigen

$a = Bc + a$ CSparse

$a = \alpha Bc + \beta a$ OSKI

Number of Variants: 3

We need a Tensor Algebra Compiler

$a = Bc$	Eigen
$a = Bc + a$	CSparse
$a = \alpha Bc + \beta a$	OSKI
$a = Bc + b$	PETSc
$a = B^T c$	PETSc
$a = B^T c + d$	PETSc

Number of Variants: 6

We need a Tensor Algebra Compiler

$a = Bc$	Eigen
$a = Bc + a$	CSparse
$a = \alpha Bc + \beta a$	OSKI
$a = Bc + b$	PETSc
$a = B^T c$	PETSc
$a = B^T c + d$	PETSc

Binary kernels are not enough

Number of Variants: 6

We need a Tensor Algebra Compiler

$a = Bc$	Eigen
$a = Bc + a$	CSparse
$a = \alpha Bc + \beta a$	OSKI
$a = Bc + b$	PETSc
$a = B^T c$	PETSc
$a = B^T c + d$	PETSc

$\times$

CSR

CSC

COO

ELL

DIA

Number of Variants: 30

We need a Tensor Algebra Compiler

$a = Bc$	Eigen
$a = Bc + a$	CSparse
$a = \alpha Bc + \beta a$	OSKI
$a = Bc + b$	PETSc
$a = B^T c$	PETSc
$a = B^T c + d$	PETSc

$\times$

- (B) CSR
- (B) CSC
- (B) COO
- (B) ELL
- (B) DIA

Number of Variants: 60

We need a Tensor Algebra Compiler

$a = Bc$	Eigen
$a = Bc + a$	CSparse
$a = \alpha Bc + \beta a$	OSKI
$a = Bc + b$	PETSc
$a = B^T c$	PETSc
$a = B^T c + d$	PETSc

$\times$

(V) (B) CSR
(V) (B) CSC
(V) (B) COO
(V) (B) ELL
(V) (B) DIA

Number of Variants: 120

We need a Tensor Algebra Compiler

$a = Bc$	Eigen
$a = Bc + a$	CSparse
$a = \alpha Bc + \beta a$	OSKI
$a = Bc + b$	PETSc
$a = B^T c$	PETSc
$a = B^T c + d$	PETSc

$\times$

(D) (V) (B) CSR
(D) (V) (B) CSC
(V) (B) COO
(V) (B) ELL
(V) (B) DIA

Number of Variants: 132

We need a Tensor Algebra Compiler

$a = Bc$	Eigen
$a = Bc + a$	CSparse
$a = \alpha Bc + \beta a$	OSKI
$a = Bc + b$	PETSc
$a = B^T c$	PETSc
$a = B^T c + d$	PETSc

$\times$

(D) (V) (B) CSR
(D) (V) (B) CSC
(V) (B) COO
(V) (B) ELL
(V) (B) DIA

$\times$

Dense Vectors
Sparse Vectors

Number of Variants: 528

We need a Tensor Algebra Compiler

$a = Bc$	Eigen
$a = Bc + a$	CSparse
$a = \alpha Bc + \beta a$	OSKI
$a = Bc + b$	PETSc
$a = B^T c$	PETSc
$a = B^T c + d$	PETSc

$\times$

(D) (V) (B) CSR
(D) (V) (B) CSC
(V) (B) COO
(V) (B) ELL
(V) (B) DIA

$\times$

Dense Vectors
Sparse Vectors

$A = B \quad a = B^T c$

$a = Bc + a \quad a = Bc + b \quad a = b + c$

$A_{ij} = \sum_k B_{ijk} * c_k \quad a = B^T c + d \quad a = \alpha Bc + \beta a$

$A = BC \quad A = 0$

$a = BCd \quad A = B + C \quad A = B \odot C \quad A = B^T \quad A = (B + C)d$

$A = \alpha B \quad A = \alpha A + \beta B \quad A = \alpha B + A \quad A = A + \alpha I \quad A = B + C + D$

$A = (B + C)(d + e) \quad A_{ik} = \sum_j B_{ijk} * c_j \quad A_{ij} = \sum_k B_{ijk} * c_k \quad A_{ilk} = \sum_j B_{ijk} * C_{lj} \quad A_{il} = \sum_{j,k} B_{ijk} * C_{jl} * D_{kl}$

$A_{jl} = \sum_{i,k} B_{ijk} * C_{il} * D_{kl} \quad A_{jl} = \sum_{i,k} B_{ijk} * C_{il} * D_{kl} \quad A_{ik} = \sum_i B_{ijk} * c_j \quad A_{jl} = \sum_{i,k} B_{ijk} * C_{il} * D_{kl} \quad A_{ijl} = \sum_k B_{ijk} * C_{lk}$

$A_{ij} = B_{ij} \sum_k (C_{ik} D_{kj}) \quad A_{ij} = \sum_k B_{ijk} C_{ijk} + D_{ij} \quad A_{ij} = \sum_k C_{ijk} D_{ijk} \quad A_{ij} = \sum_{l,k} B_{lik} * C_{lj} * D_{kj} \quad A_{ij} = \sum_{k,l} B_{ikjl} * C_{kl}$

$A_{ij} = B_{ij}(C_{ik} D_{kj})$

Infinite number of possible tensor formats and expressions

Number of Variants: ∞

Advantages of a Tensor Algebra Compiler

Traditional Libraries

Choose a few expressions

Choose a few formats

Spend years hand optimizing kernels

The Tensor Algebra Compiler (taco)

Any expression

Many formats

Compiler produces optimized code

Advantages of a Tensor Algebra Compiler

Traditional Libraries

Choose a few expressions

Choose a few formats

Spend years hand optimizing kernels

$$A = B + C \qquad a = b + c$$
$$A = BC \qquad a = \alpha Bc + \beta a$$
$$A = \alpha B \qquad A = B^T$$

The Tensor Algebra Compiler (taco)

Any expression

Many formats

Compiler produces optimized code

Advantages of a Tensor Algebra Compiler

Traditional Libraries

Choose a few expressions

Choose a few formats

Spend years hand optimizing kernels

$$A = B + C \quad a = b + c \quad A = BC \quad a = \alpha Bc + \beta a \quad A = \alpha B \quad A = B^T$$

The Tensor Algebra Compiler (taco)

Any expression

Many formats

Compiler produces optimized code

$$a = Bc \quad A = B \quad a = Bc + a \quad a = Bc + b \quad A_{ij} = \sum_k B_{ijk} * c_k \quad a = B^T c \quad a = b + c \quad a = \alpha Bc + \beta a \quad A = B + C \quad a = B^T c + d \quad A = B \odot C \quad A = \alpha B + A \quad A = 0 \quad A = BC \quad A = B^T \quad A = \alpha A + \beta B \quad A = \alpha B \quad A = A + \alpha I \quad A_{ik} = \sum_j B_{ijk} * c_j \quad A = (B + C)d \quad A_{il} = \sum_{j,k} B_{ijk} * C_{jl} * D_{kl} \quad A = B + C + D \quad a = BCd \quad A = (B + C)(d + e) \quad A_{jl} = \sum_{i,k} B_{ijk} * C_{il} * D_{kl} \quad A_{ijl} = \sum_k B_{ijk} * C_{lk} \quad A_{ilk} = \sum_j B_{ijk} * C_{lj}$$

Advantages of a Tensor Algebra Compiler

Traditional Libraries	The Tensor Algebra Compiler (taco)
Choose a few expressions	Any expression
Choose a few formats	Many formats
Spend years hand optimizing kernels	Compiler produces optimized code

CSR

CSC

Advantages of a Tensor Algebra Compiler

Traditional Libraries

Choose a few expressions

Choose a few formats

Spend years hand optimizing kernels

CSR

CSC

The Tensor Algebra Compiler (taco)

Any expression

Many formats

Compiler produces optimized code

DCSR

DCSR

BCSC

DCSC

CSR

CSB

CSC

BCSR

Sparse Vector

BCSC

Dense Tensor

Sparse Tensors

Advantages of a Tensor Algebra Compiler

Traditional Libraries	The Tensor Algebra Compiler (taco)
Choose a few expressions	Any expression
Choose a few formats	Many formats
Spend years hand optimizing kernels	Compiler produces optimized code

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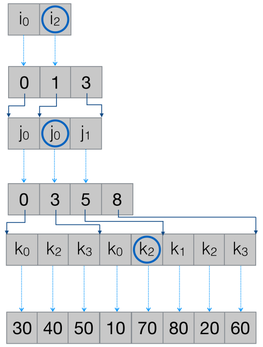
Sparse code and data

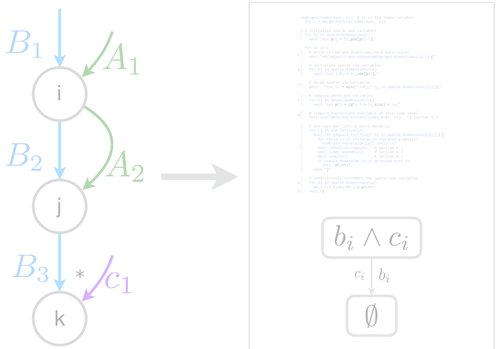



```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

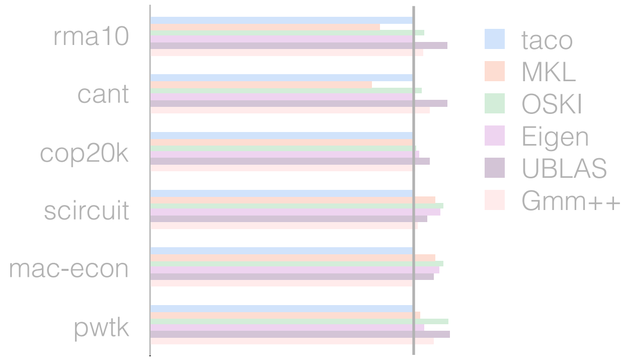
  for (int pB2 = B2_pos[pB1];
           pB2 < B2_pos[pB1 + 1];
           pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        a[pA2] += b[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

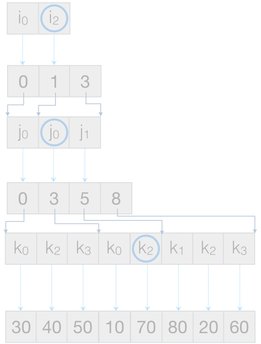
Intermediate Representation and Code Generation



Evaluation



Sparse code and data

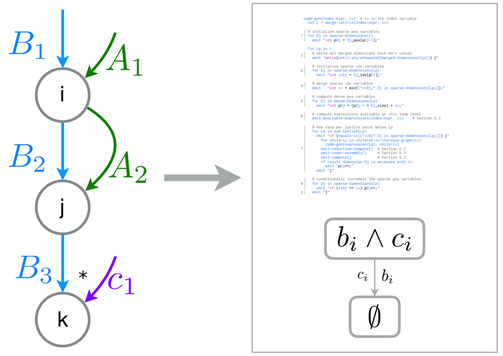



```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

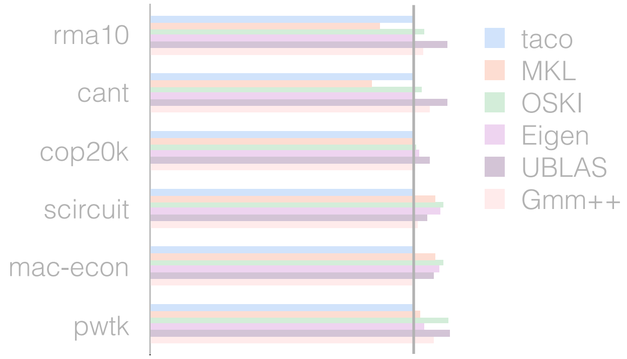
  for (int pB2 = B2_pos[pB1];
            pB2 < B2_pos[pB1 + 1];
            pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        a[pA2] += b[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

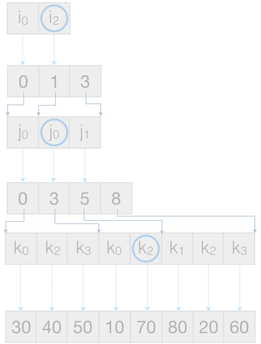
Intermediate Representation and Code Generation



Evaluation



Sparse code and data




```
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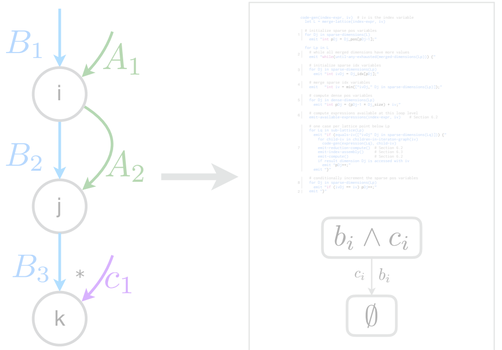
  int i = B1_idx[pB1];

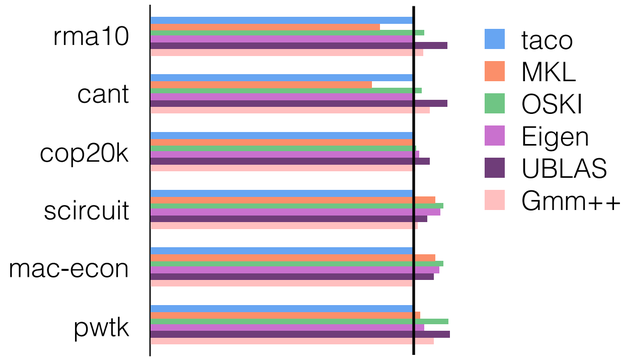
  for (int pB2 = B2_pos[pB1];
           pB2 < B2_pos[pB1 + 1];
           pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        a[pA2] += b[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

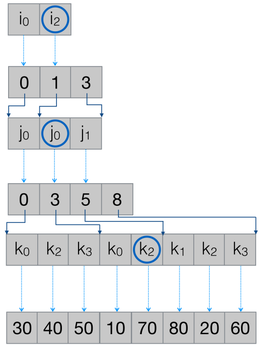
Intermediate Representation and Code Generation



Evaluation



Sparse code and data

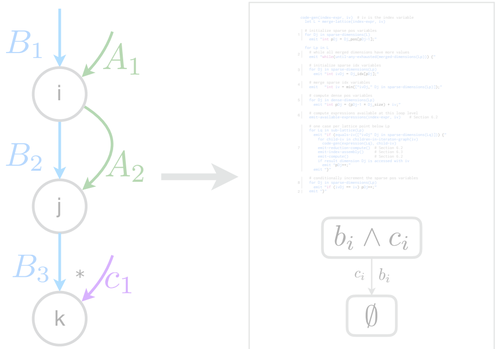



```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

  for (int pB2 = B2_pos[pB1];
           pB2 < B2_pos[pB1 + 1];
           pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        a[pA2] += b[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

Intermediate Representation and Code Generation



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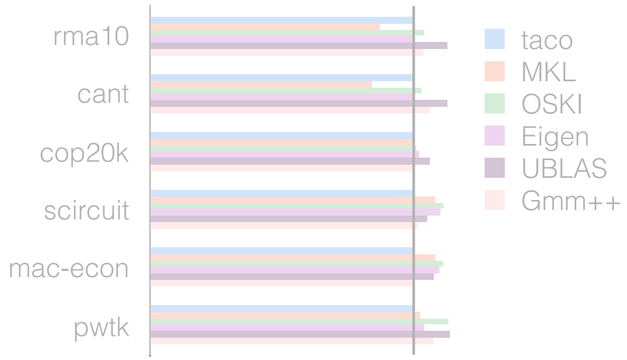


Tensor-vector multiplication example

$$A_{ij} = \sum_{k} B_{ijk} * c_k$$

Tensor-vector multiplication example

Free variables i, j

$$A_{ij} = \sum_{k} B_{ijk} * c_k$$

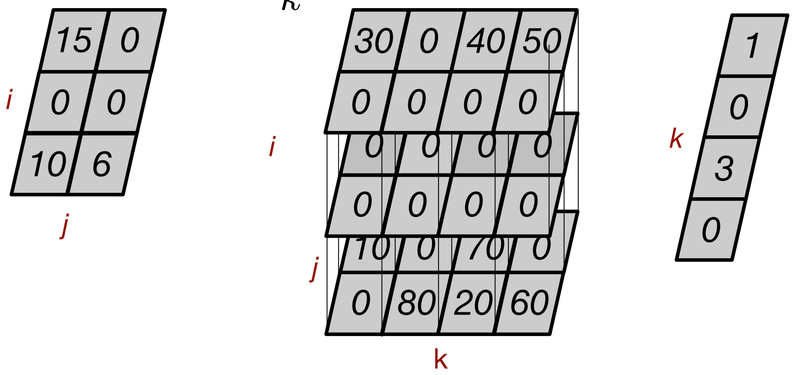
Tensor-vector multiplication example

Summation variable k

$$A_{ij} = \sum_{k} B_{ijk} * c_{k}$$

Tensor-vector multiplication example

$$A_{ij} = \sum_k B_{ijk} * c_k$$



(dense, dense) (dense,dense,dense) (dense)

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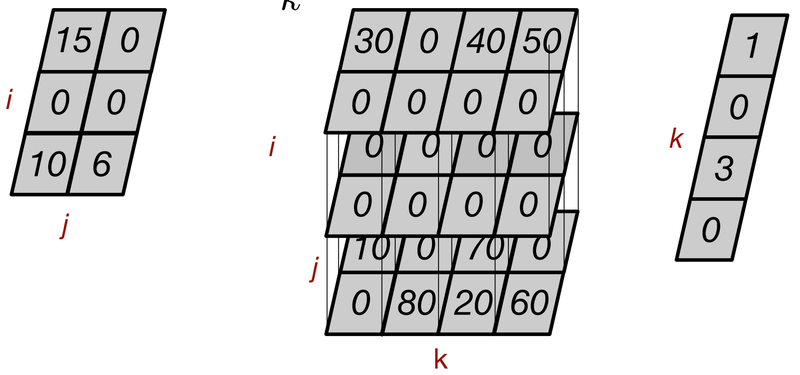


(dense, dense) (dense,dense,dense) (dense)

```
for (int i = 0; i < m; i++) {


  for (int j = 0; j < n; j++) {
    int pB2 = (i * n) + j;
    int pA2 = (i * n) + j;



    for (int k = 0; k < p; k++) {
      int pB3 = (pB2 * p) + k;



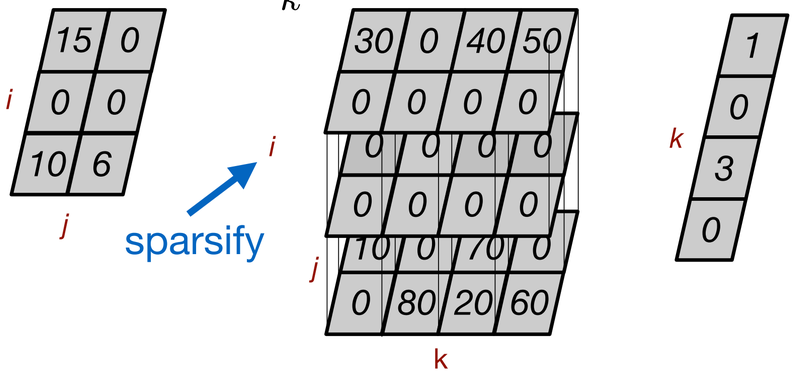
      a[pA2] += b[pB3] * c[k];



    }
  }
}
```

Tensor-vector multiplication example

$$A_{ij} = \sum_k B_{ijk} * c_k$$



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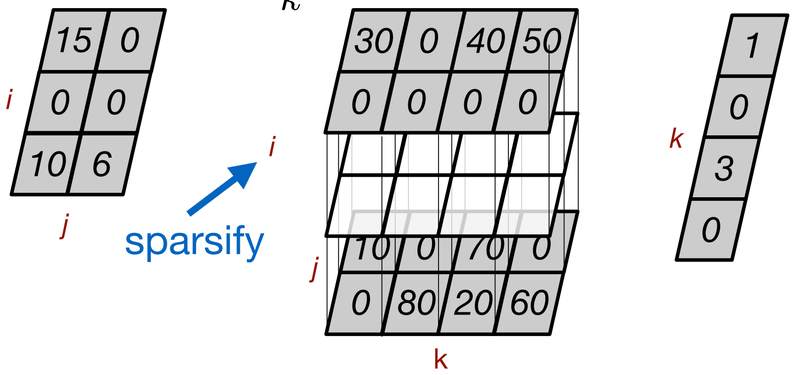

(dense, dense) (sparse,dense,dense) (dense)

```
for (int i = 0; i < m; i++) {

  for (int j = 0; j < n; j++) {
    int pB2 = (i * n) + j;
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      int pB3 = (pB2 * p) + k;


      a[pA2] += b[pB3] * c[k];


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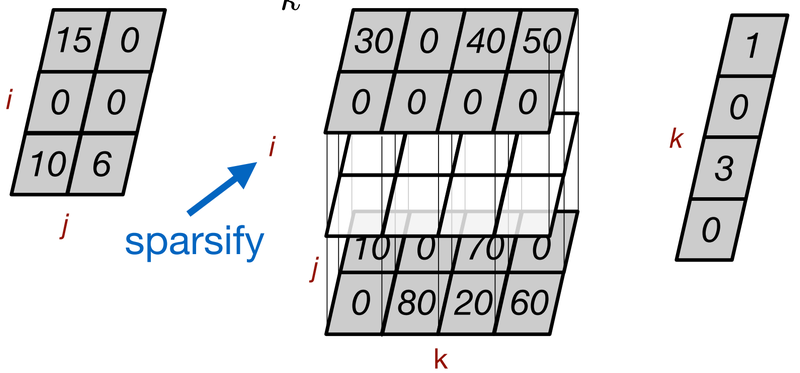

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    for (int k = 0; k < p; k++) {
      int pB3 = (pB2 * p) + k;


      a[pA2] += b[pB3] * c[k];


    }
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```

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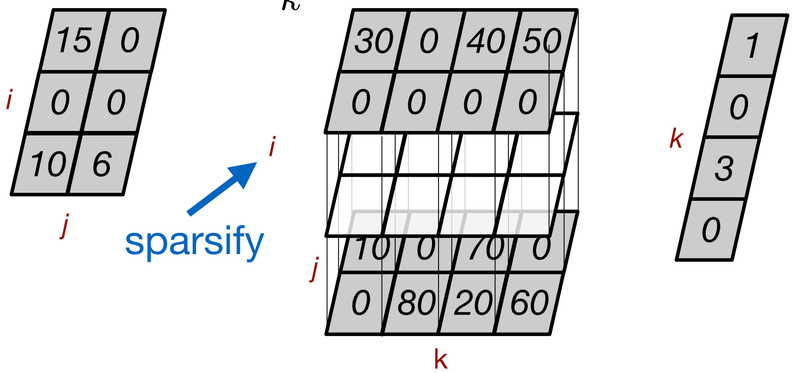

(dense, dense) (sparse,dense,dense) (dense)

```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

  for (int j = 0; j < n; j++) {
    int pB2 = (pB1 * n) + j;
    int pA2 = (i * n) + j;



    for (int k = 0; k < p; k++) {
      int pB3 = (pB2 * p) + k;



      a[pA2] += b[pB3] * c[k];



    }
  }
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```

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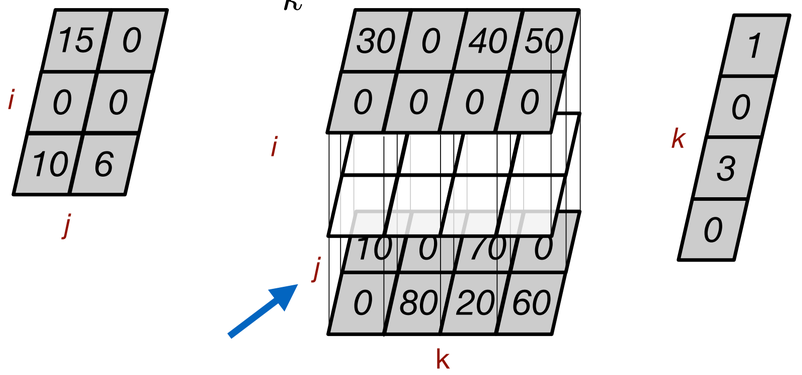

(dense, dense) (sparse,dense,dense) (dense)

```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
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    int pA2 = (i * n) + j;


    for (int k = 0; k < p; k++) {
      int pB3 = (pB2 * p) + k;


      a[pA2] += b[pB3] * c[k];


    }
  }
}
```

Tensor-vector multiplication example

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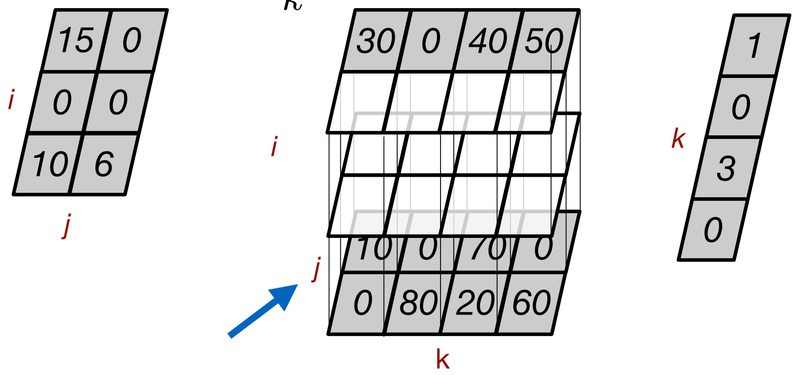


(dense, dense) (sparse,sparse,dense) (dense)

```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

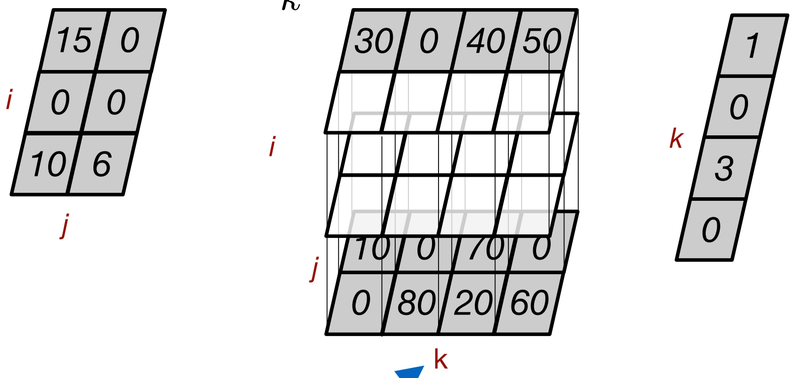

    for (int k = 0; k < p; k++) {
      int pB3 = (pB2 * p) + k;


      a[pA2] += b[pB3] * c[k];


    }
  }
}
```

Tensor-vector multiplication example

$$A_{ij} = \sum_k B_{ijk} * c_k$$



(dense, dense) (sparse,sparse,dense) (dense)

```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;


    for (int k = 0; k < p; k++) {
      int pB3 = (pB2 * p) + k;


      a[pA2] += b[pB3] * c[k];


    }
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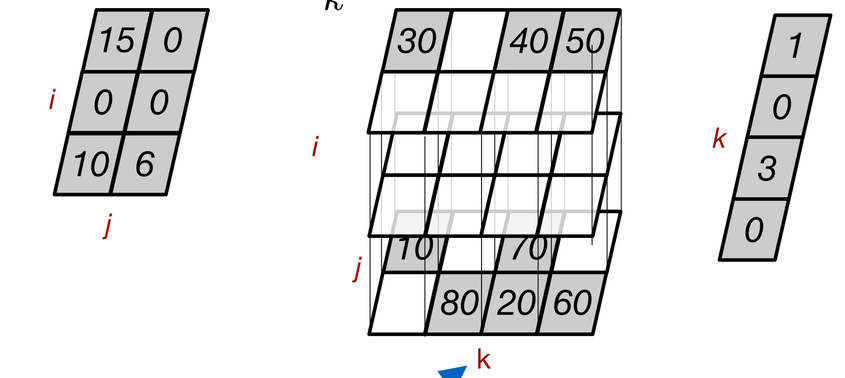
```

Tensor-vector multiplication example

$$A_{ij} = \sum_k B_{ijk} * c_k$$




```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

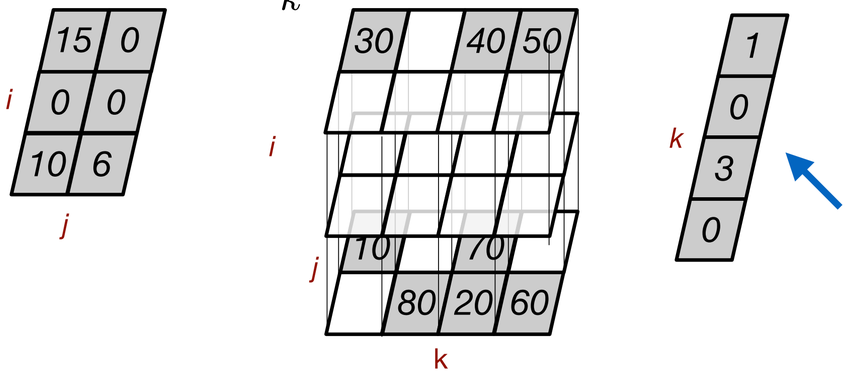

    for (int pB3 = B3_pos[pB2]; pB3 < B3_pos[pB2 + 1]; pB3++) {
      int k = B3_idx[pB3];


      a[pA2] += b[pB3] * c[k];


    }
  }
}
```

Tensor-vector multiplication example

$$A_{ij} = \sum_k B_{ijk} * c_k$$



(dense, dense) (sparse,sparse,sparse) (dense)

```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;



    for (int pB3 = B3_pos[pB2]; pB3 < B3_pos[pB2 + 1]; pB3++) {
      int k = B3_idx[pB3];



      a[pA2] += b[pB3] * c[k];



    }
  }
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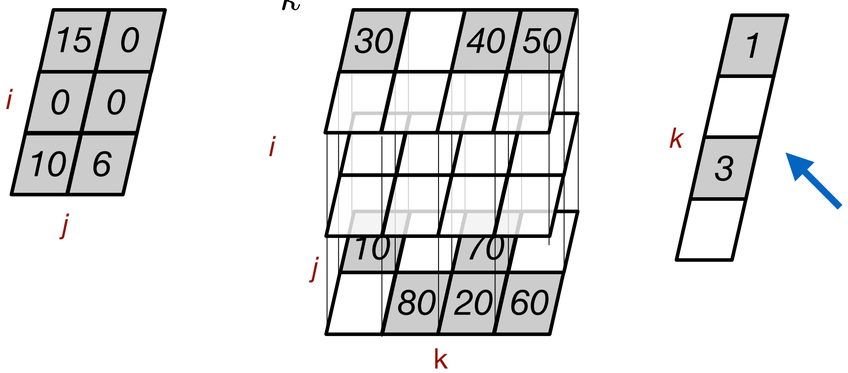
Tensor-vector multiplication example

$$A_{ij} = \sum_k B_{ijk} * c_k$$



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```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;



    for (int pB3 = B3_pos[pB2]; pB3 < B3_pos[pB2 + 1]; pB3++) {
      int k = B3_idx[pB3];



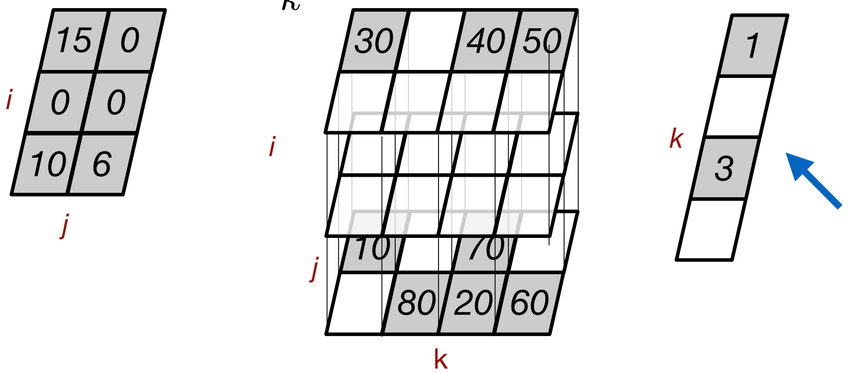
      a[pA2] += b[pB3] * c[k];



    }
  }
}
```

Tensor-vector multiplication example

$$A_{ij} = \sum_k B_{ijk} * c_k$$



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```
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    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    for (int pB3 = B3_pos[pB2]; pB3 < B3_pos[pB2 + 1]; pB3++) {
      int k = B3_idx[pB3];

      a[pA2] += b[pB3] * c[k];

    }
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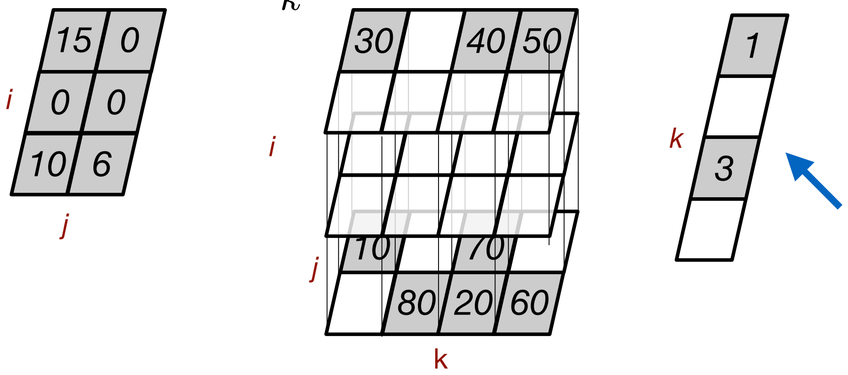
Tensor-vector multiplication example

$$A_{ij} = \sum_k B_{ijk} * c_k$$



(dense, dense) (sparse,sparse,sparse) (sparse)

```
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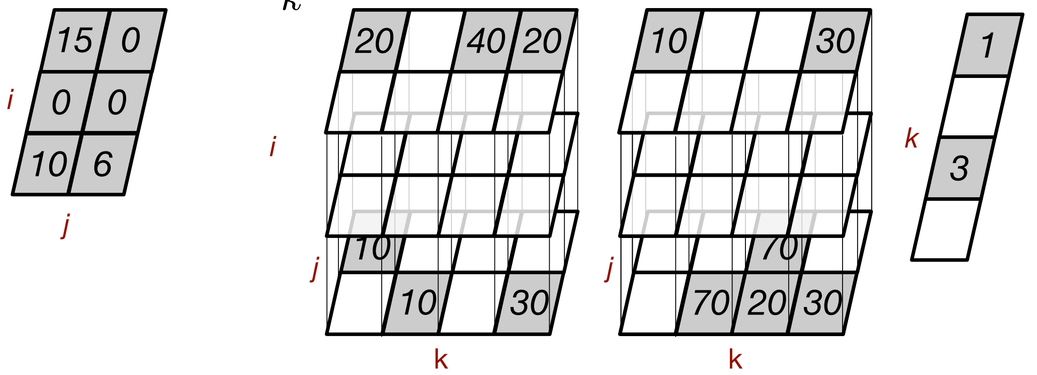
  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        a[pA2] += b[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

Tensor-vector multiplication example

$$A_{ij} = \sum_k (B_{ijk} + D_{ijk}) * c_k$$



(dense, dense)

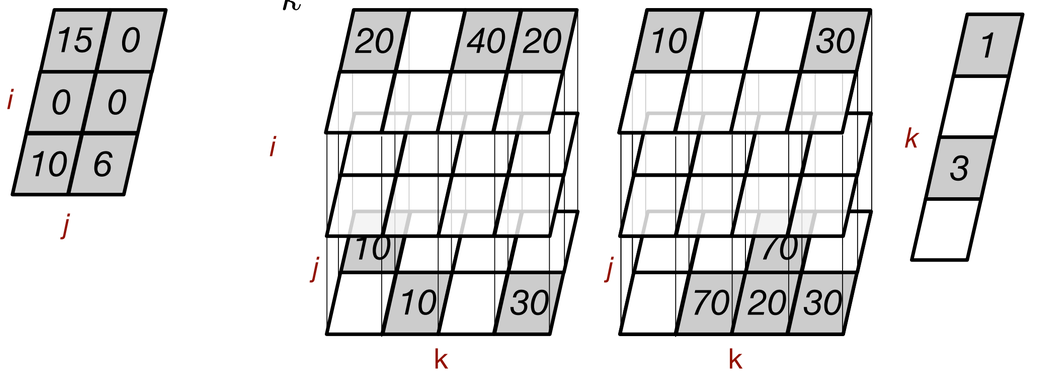
(sparse,sparse,sparse)

(sparse,sparse,sparse)

(sparse)

Tensor-vector multiplication example

$$A_{ij} = \sum_k (B_{ijk} + D_{ijk}) * c_k$$



(dense, dense)

(sparse,sparse,sparse)

(sparse,sparse,sparse)

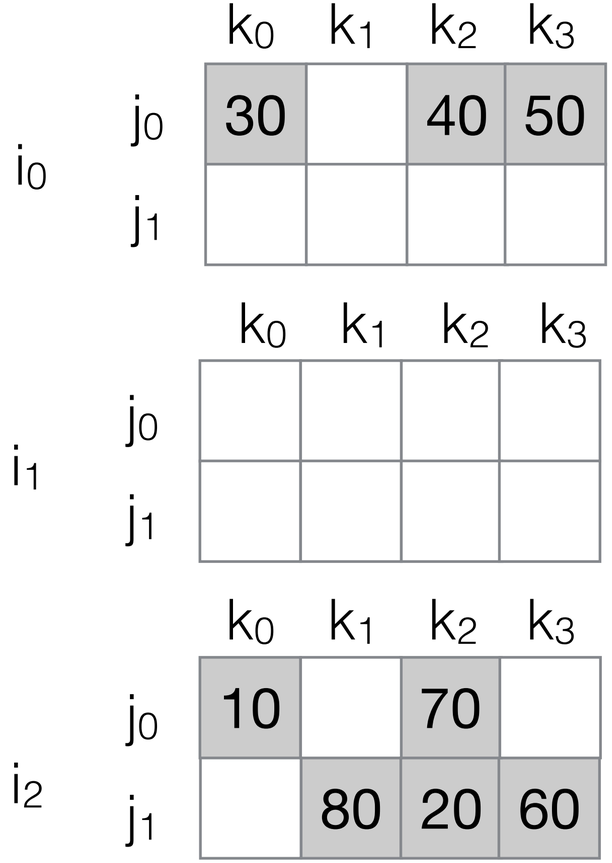
(sparse)

```
int pB1 = B1_pos[0];
int pD1 = D1_pos[0];
while (pB1 < B1_pos[1] &&
       pD1 < D1_pos[1]) {
  int iB = B1_idx[pB1];
  int iD = D1_idx[pD1];
  int i = min(iB, iD);
  if (iB == i && iD == i) {
    int pB2 = B2_pos[pB1];
    int pD2 = D2_pos[pD1];
    while (pB2 < B2_pos[pB1 + 1] &&
           pD2 < D2_pos[pD1 + 1]) {
      int jB = B2_idx[pB2];
      int jD = D2_idx[pD2];
      int j = min(jB, jD);
      int pA2 = (i * A2_dimension) + j;
      if (jB == j && jD == j) {
        double tk = 0.0;
        int pB3 = B3_pos[pB2];
        int pD3 = D3_pos[pD2];
        int pc1 = c1_pos[0];
        while (pB3 < B3_pos[pB2 + 1] &&
               pD3 < D3_pos[pD2 + 1] &&
               pc1 < c1_pos[1]) {
          int kB = B3_idx[pB3];
          int kD = D3_idx[pD3];
          int kc = c1_idx[pc1];
          int k = min(kB, kD, kc);
          if (kB == k && kD == k && kc == k) {
            tk += (b[pB3] + d[pD3]) * c[pc1];
          }
          else if (kB == k && kc == k) {
            tk += b[pB3] * c[pc1];
          }
          else if (kD == k && kc == k) {
            tk += d[pD3] * c[pc1];
          }
          if (kB == k) pB3++;
          if (kD == k) pD3++;
          if (kc == k) pc1++;
        }
        while (pB3 < B3_pos[pB2 + 1] &&
               pc1 < c1_pos[1]) {
          int kB0 = B3_idx[pB3];
          int kc0 = c1_idx[pc1];
          int k0 = min(kB0, kc0);
          if (kB0 == k0 && kc0 == k0) {
            tk += b[pB3] * c[pc1];
          }
          if (kB0 == k0) pB3++;
          if (kc0 == k0) pc1++;
        }
        while (pD3 < D3_pos[pD2 + 1] &&
               pc1 < c1_pos[1]) {
          int kD0 = D3_idx[pD3];
          int kc1 = c1_idx[pc1];
          int k1 = min(kD0, kc1);
          if (kD0 == k1 && kc1 == k1) {
            tk += d[pD3] * c[pc1];
          }
          if (kD0 == k1) pD3++;
          if (kc1 == k1) pc1++;
        }
        a[pA2] = tk;
      }
      else if (jB == j) {
        double tk0 = 0.0;
        int pB30 = B3_pos[pB2];
        int pc10 = c1_pos[0];
        while (pB30 < B3_pos[pB2 + 1] &&
               pc10 < c1_pos[1]) {
          int kB1 = B3_idx[pB30];
          int kc2 = c1_idx[pc10];
          int k2 = min(kB1, kc2);
          if (kB1 == k2 && kc2 == k2) {
            tk0 += b[pB30] * c[pc10];
          }
          if (kB1 == k2) pB30++;
          if (kc2 == k2) pc10++;
        }
        a[pA2] = tk0;
      }
      else {
        double tk1 = 0.0;
        int pD30 = D3_pos[pD2];
        int pc11 = c1_pos[0];
        while (pD30 < D3_pos[pD2 + 1] &&
               pc11 < c1_pos[1]) {
          int kD1 = D3_idx[pD30];
          int kc3 = c1_idx[pc11];
          int k3 = min(kD1, kc3);
          if (kD1 == k3 && kc3 == k3) {
            tk1 += d[pD30] * c[pc11];
          }
          if (kD1 == k3) pD30++;
          if (kc3 == k3) pc11++;
        }
        a[pA2] = tk1;
      }
      if (jB == j) pB2++;
      if (jD == j) pD2++;
    }
```

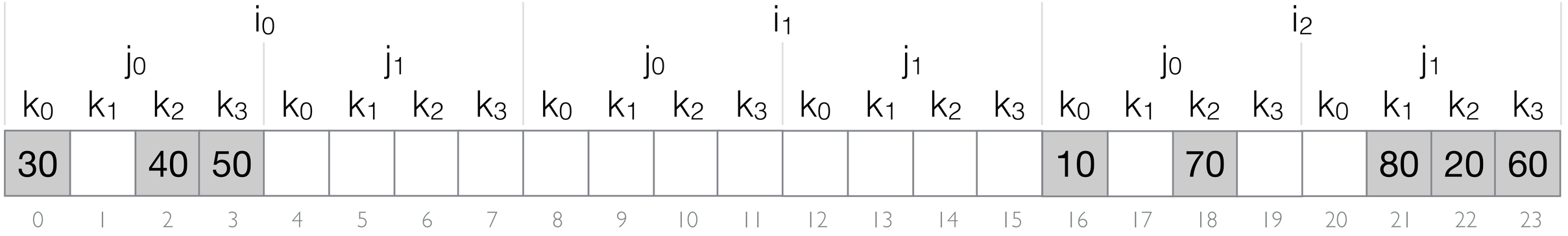
```
    while (pB2 < B2_pos[pB1 + 1]) {
      int jB0 = B2_idx[pB2];
      int pA20 = (i * A2_dimension) + jB0;
      double tk2 = 0.0;
      int pB31 = B3_pos[pB2];
      int pc12 = c1_pos[0];
      while (pB31 < B3_pos[pB2 + 1] &&
             pc12 < c1_pos[1]) {
        int kB2 = B3_idx[pB31];
        int kc4 = c1_idx[pc12];
        int k4 = min(kB2, kc4);
        if (kB2 == k4 && kc4 == k4) {
          tk2 += b[pB31] * c[pc12];
        }
        if (kB2 == k4) pB31++;
        if (kc4 == k4) pc12++;
      }
      a[pA20] = tk2;
      pB2++;
    }
    while (pD2 < D2_pos[pD1 + 1]) {
      int jD0 = D2_idx[pD2];
      int pA21 = (i * A2_dimension) + jD0;
      double tk3 = 0.0;
      int pD31 = D3_pos[pD2];
      int pc13 = c1_pos[0];
      while (pD31 < D3_pos[pD2 + 1] &&
             pc13 < c1_pos[1]) {
        int kD2 = D3_idx[pD31];
        int kc5 = c1_idx[pc13];
        int k5 = min(kD2, kc5);
        if (kD2 == k5 && kc5 == k5) {
          tk3 += d[pD31] * c[pc13];
        }
        if (kD2 == k5) pD31++;
        if (kc5 == k5) pc13++;
      }
      a[pA21] = tk3;
      pD2++;
    }
  }
  else if (iB == i) {
    for (int pB2 = B2_pos[pB1];
                pB2 < B2_pos[pB1 + 1];
                pB2++) {
      int jB1 = B2_idx[pB2];
      int pA22 = (i * A2_dimension) + jB1;
      double tk4 = 0.0;
      int pB32 = B3_pos[pB2];
      int pc14 = c1_pos[0];
      while (pB32 < B3_pos[pB2 + 1] &&
             pc14 < c1_pos[1]) {
        int kB3 = B3_idx[pB32];
        int kc6 = c1_idx[pc14];
        int k6 = min(kB3, kc6);
        if (kB3 == k6 && kc6 == k6) {
          tk4 += b[pB32] * c[pc14];
        }
        if (kB3 == k6) pB32++;
        if (kc6 == k6) pc14++;
      }
      a[pA22] = tk4;
    }
  }
  else {
    for (int pD2 = D2_pos[pD1];
                pD2 < D2_pos[pD1 + 1];
                pD2++) {
      int jD1 = D2_idx[pD2];
      int pA23 = (i * A2_dimension) + jD1;
      double tk5 = 0;
      int pD32 = D3_pos[pD2];
      int pc15 = c1_pos[0];
      while (pD32 < D3_pos[pD2 + 1] &&
             pc15 < c1_pos[1]) {
        int kD3 = D3_idx[pD32];
        int kc7 = c1_idx[pc15];
        int k7 = min(kD3, kc7);
        if (kD3 == k7 && kc7 == k7) {
          tk5 += d[pD32] * c[pc15];
        }
        if (kD3 == k7) pD32++;
        if (kc7 == k7) pc15++;
      }
      a[pA23] = tk5;
    }
  }
  if (iB == i) pB1++;
  if (iD == i) pD1++;
}
```

```
while (pB1 < B1_pos[1]) {
  int iB0 = B1_idx[pB1];
  for (int pB2 = B2_pos[pB1];
              pB2 < B2_pos[pB1 + 1];
              pB2++) {
    int jB2 = B2_idx[pB2];
    int pA24 = (iB0 * A2_dimension) + jB2;
    double tk6 = 0.0;
    int pB33 = B3_pos[pB2];
    int pc16 = c1_pos[0];
    while (pB33 < B3_pos[pB2 + 1] &&
           pc16 < c1_pos[1]) {
      int kB4 = B3_idx[pB33];
      int kc8 = c1_idx[pc16];
      int k8 = min(kB4, kc8);
      if (kB4 == k8 && kc8 == k8) {
        tk6 += b[pB33] * c[pc16];
      }
      if (kB4 == k8) pB33++;
      if (kc8 == k8) pc16++;
    }
    a[pA24] = tk6;
  }
  pB1++;
}
while (pD1 < D1_pos[1]) {
  int iD0 = D1_idx[pD1];
  for (int pD2 = D2_pos[pD1];
              pD2 < D2_pos[pD1 + 1];
              pD2++) {
    int jD2 = D2_idx[pD2];
    int pA25 = (iD0 * A2_dimension) + jD2;
    double tk7 = 0.0;
    int pD33 = D3_pos[pD2];
    int pc17 = c1_pos[0];
    while (pD33 < D3_pos[pD2 + 1] &&
           pc17 < c1_pos[1]) {
      int kD4 = D3_idx[pD33];
      int kc9 = c1_idx[pc17];
      int k9 = min(kD4, kc9);
      if (kD4 == k9 && kc9 == k9) {
        tk7 += d[pD33] * c[pc17];
      }
      if (kD4 == k9) pD33++;
      if (kc9 == k9) pc17++;
    }
    a[pA25] = tk7;
  }
  pD1++;
}
```

Dense storage formats



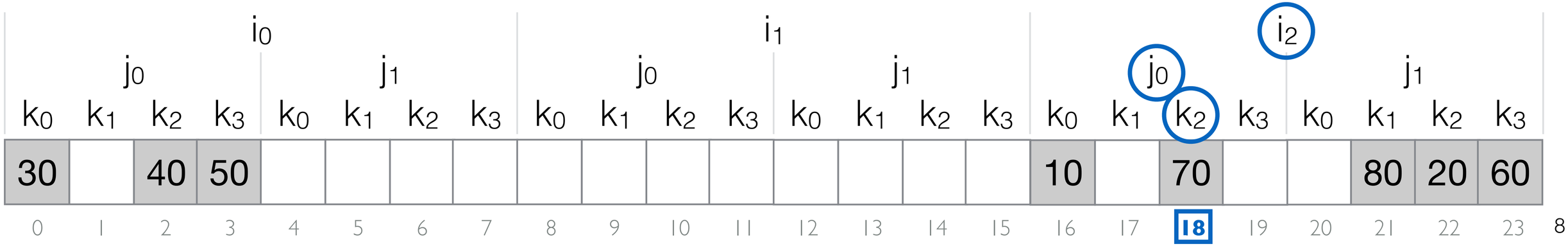
Dense storage formats



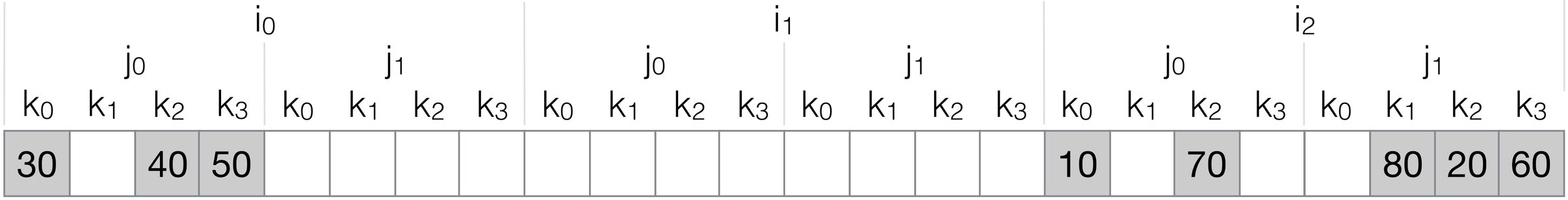
Dense storage formats

`locate (2, 0, 2)` → `i*2*4 + j*4 + k = 18`

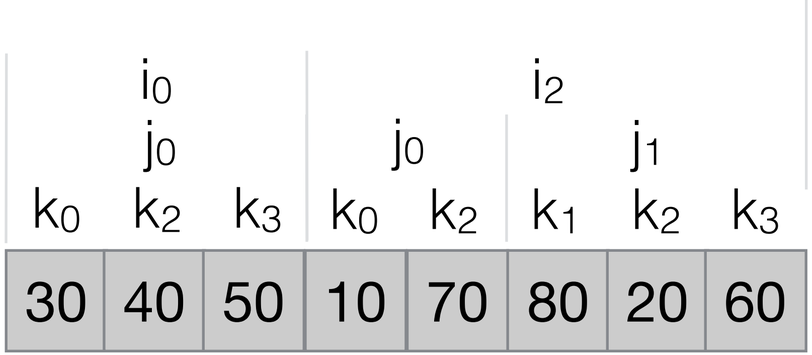
Random access: easy to locate element, but wasted storage



Compressed storage formats



Compressed storage formats

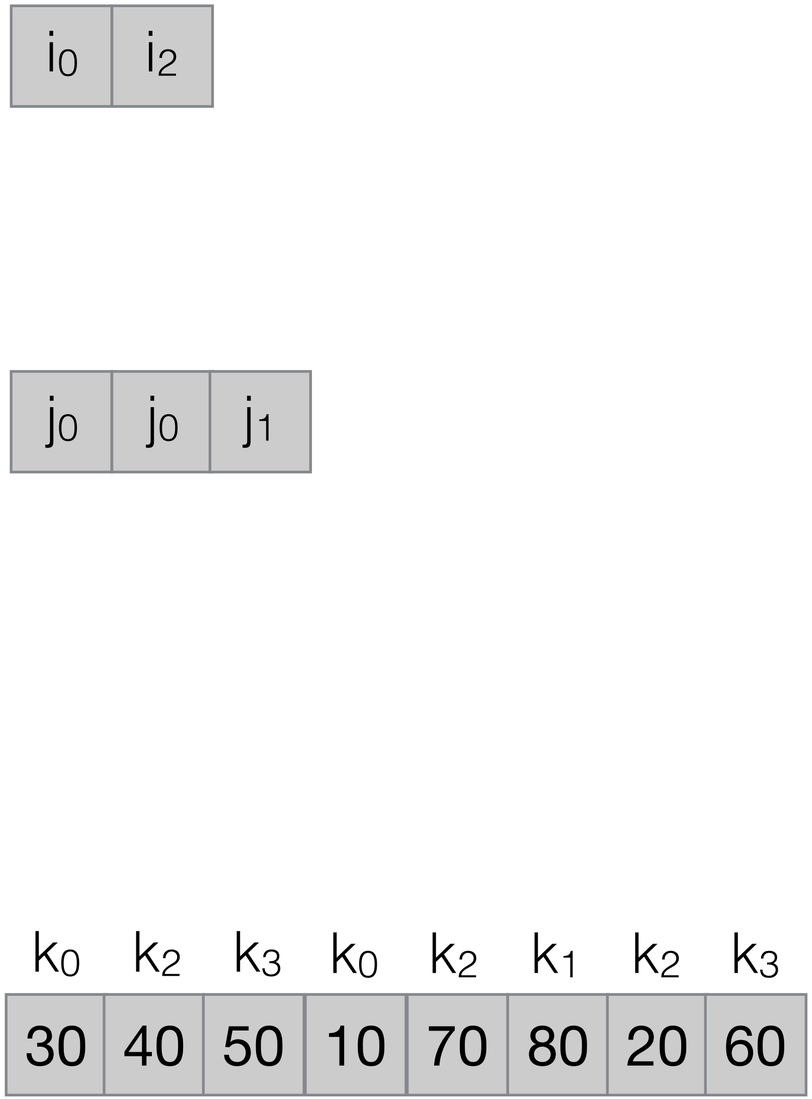


Compressed storage formats

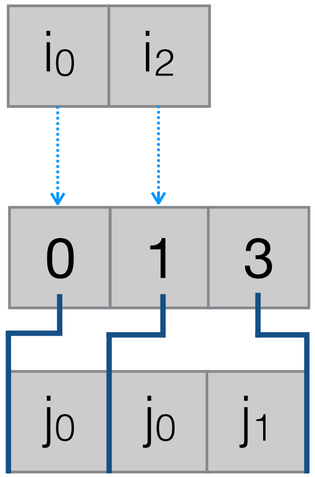
i_0	i_2

	j_0		j_0		j_1		
k_0	k_2	k_3	k_0	k_2	k_1	k_2	k_3
30	40	50	10	70	80	20	60

Compressed storage formats



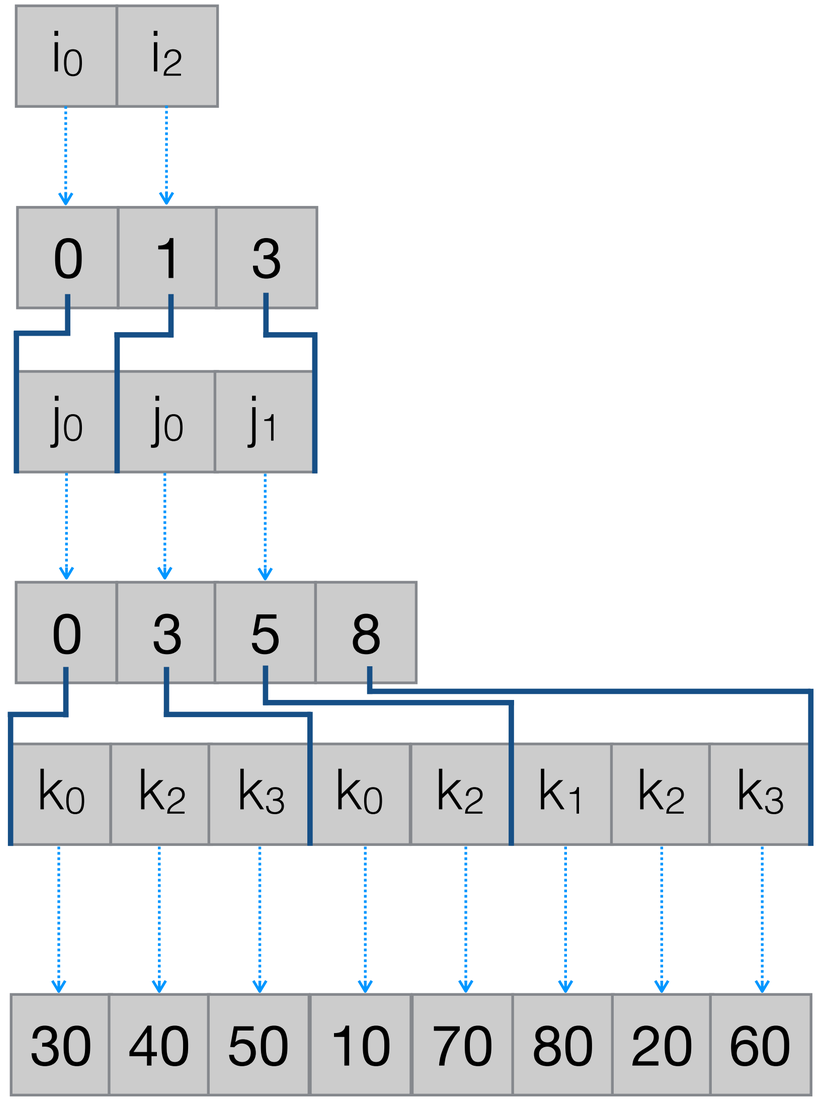
Compressed storage formats



k_0	k_2	k_3	k_0	k_2	k_1	k_2	k_3
30	40	50	10	70	80	20	60

Compressed storage formats

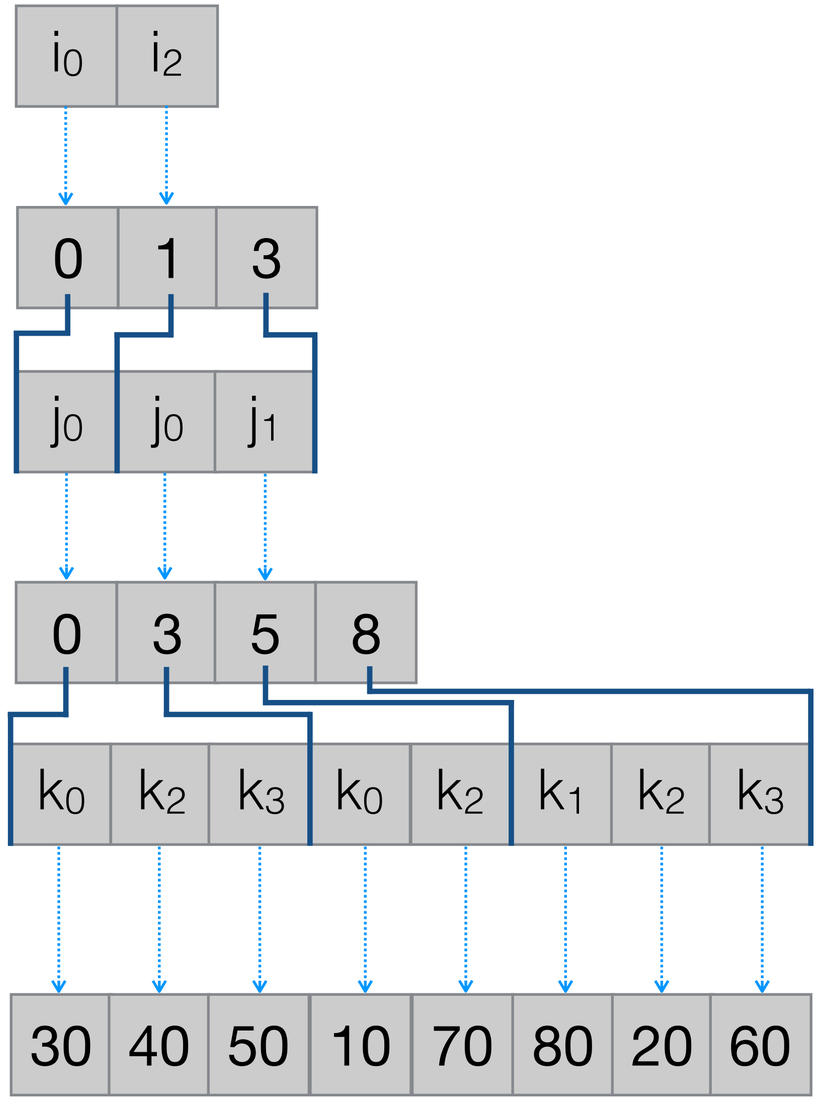
No random access: must traverse level by level to locate coordinate



Compressed storage formats

locate (i_2, j_0, k_2)

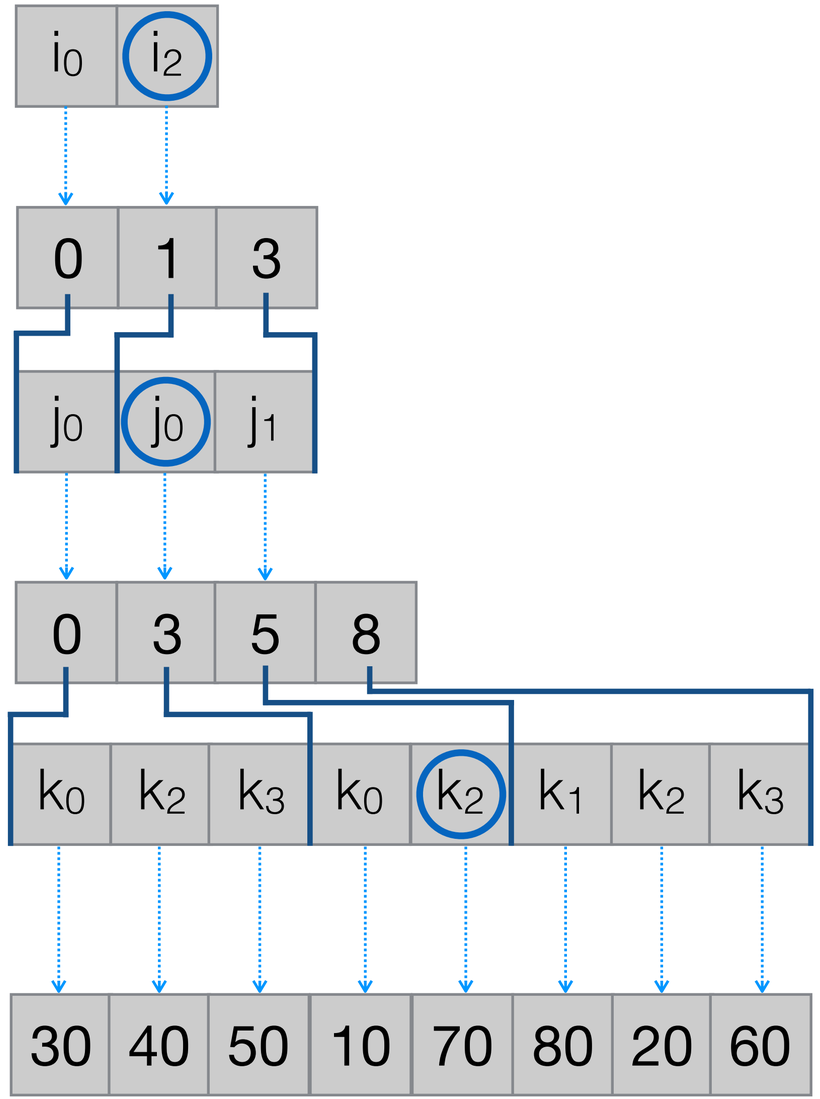
No random access: must traverse level by level to locate coordinate



Compressed storage formats

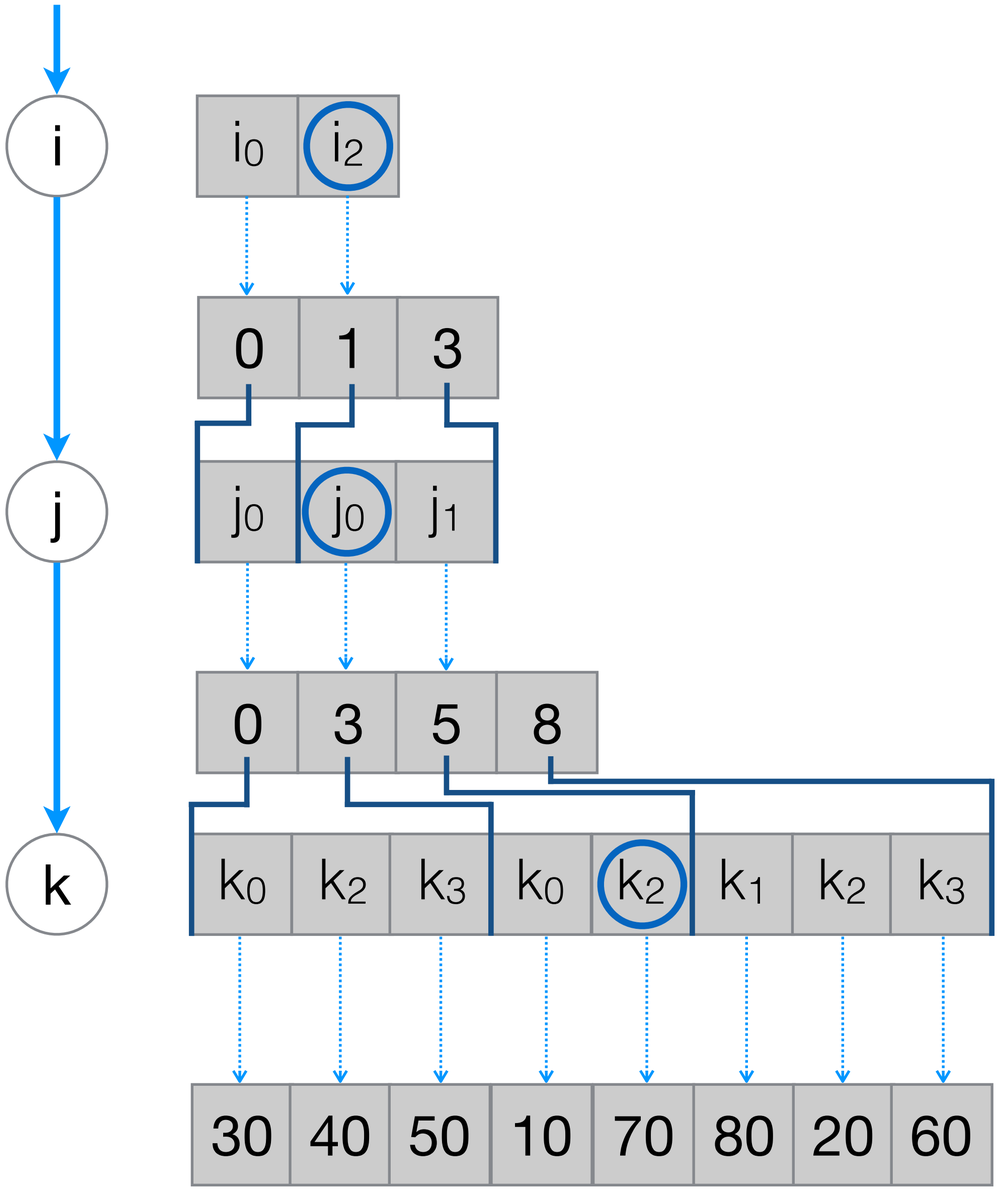
locate (i_2, j_0, k_2)

No random access: must traverse level by level to locate coordinate



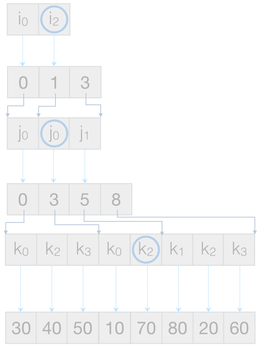
Compressed storage formats

locate (i_2, j_0, k_2)



No random access: must traverse level by level to locate coordinate

Sparse code and data

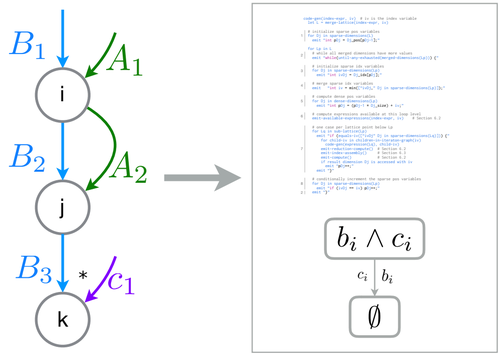



```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

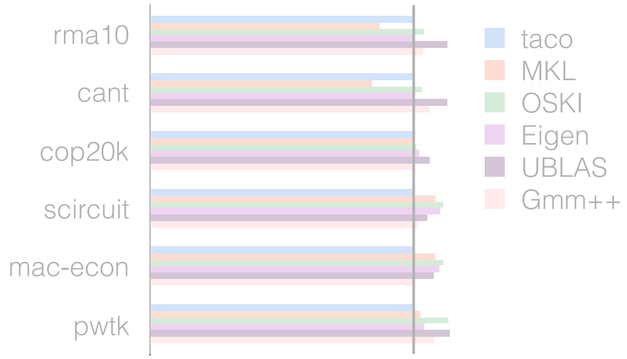
  for (int pB2 = B2_pos[pB1];
            pB2 < B2_pos[pB1 + 1];
            pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        a[pA2] += b[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

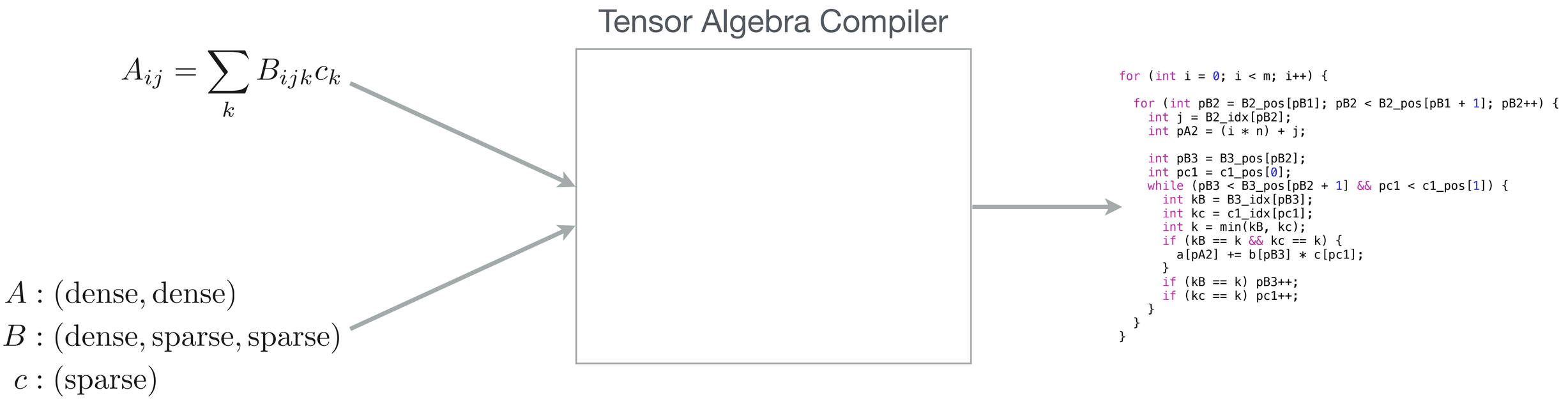
Intermediate Representation and Code Generation



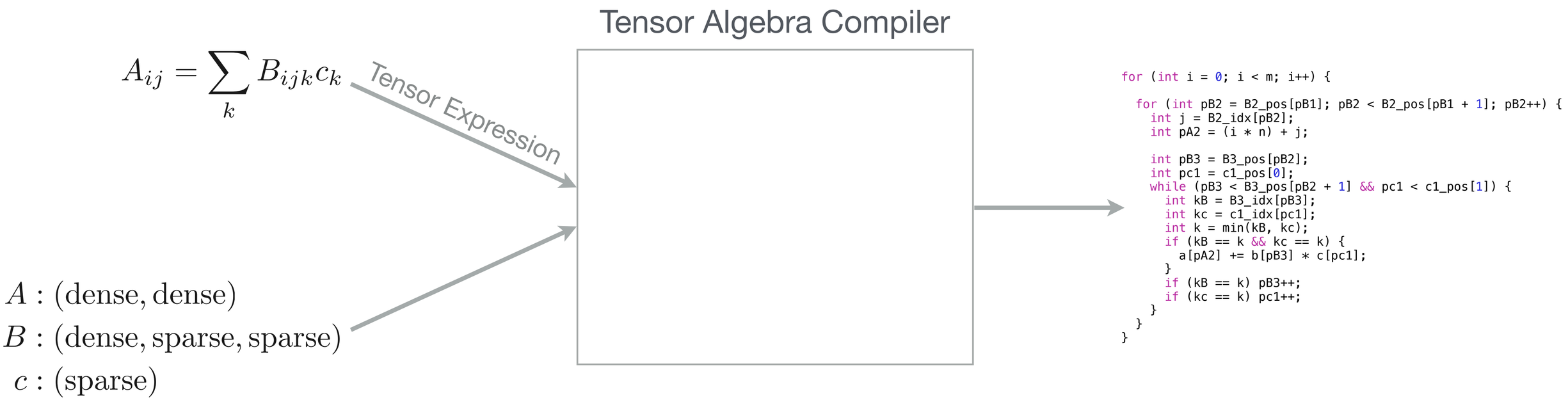
Evaluation



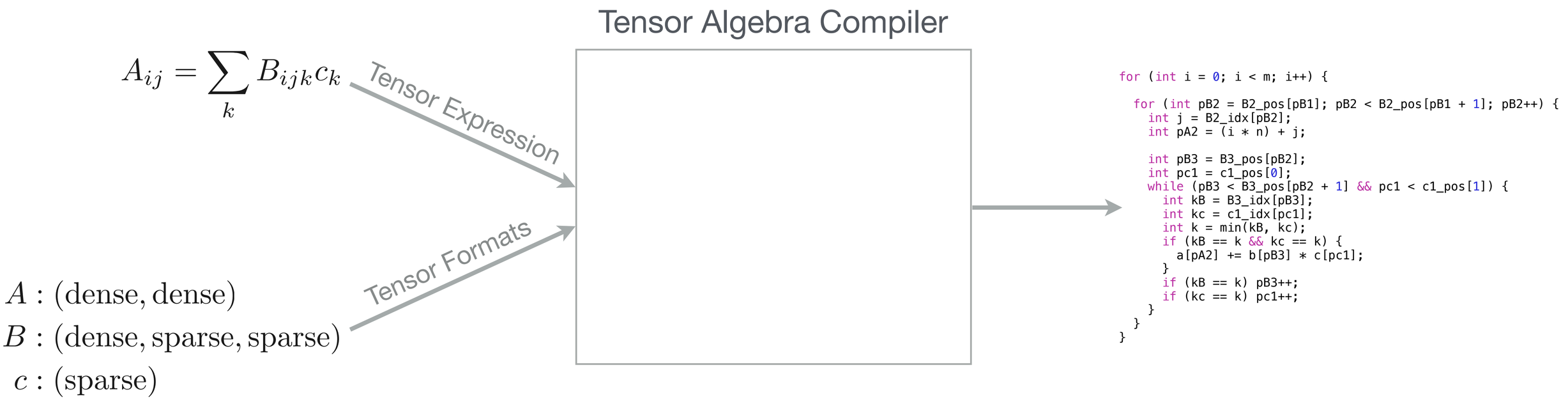
The Tensor Algebra Compiler (taco)



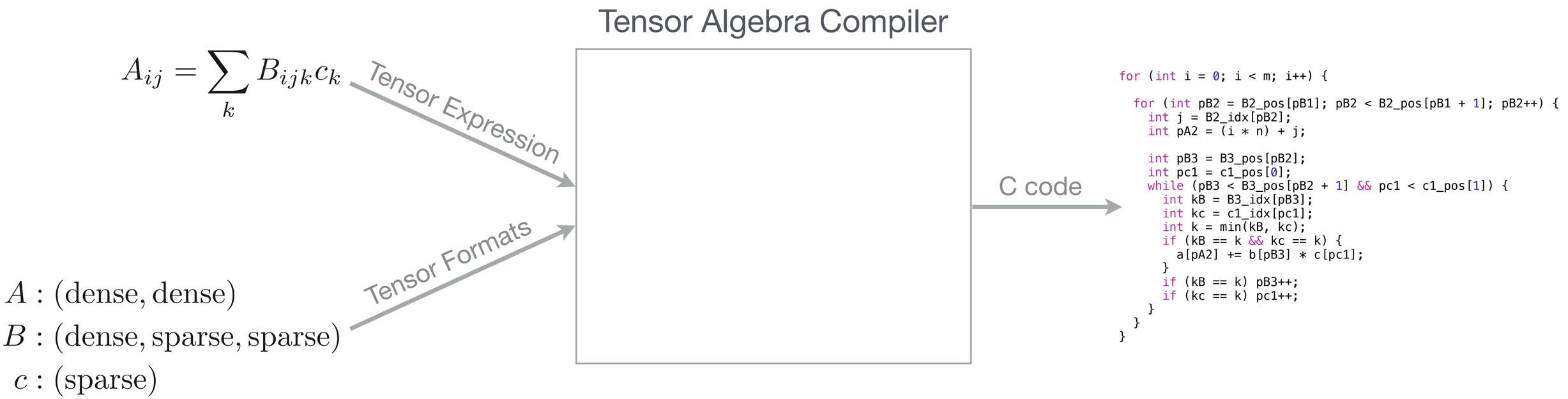
The Tensor Algebra Compiler (taco)



The Tensor Algebra Compiler (taco)



The Tensor Algebra Compiler (taco)



The Tensor Algebra Compiler (taco)

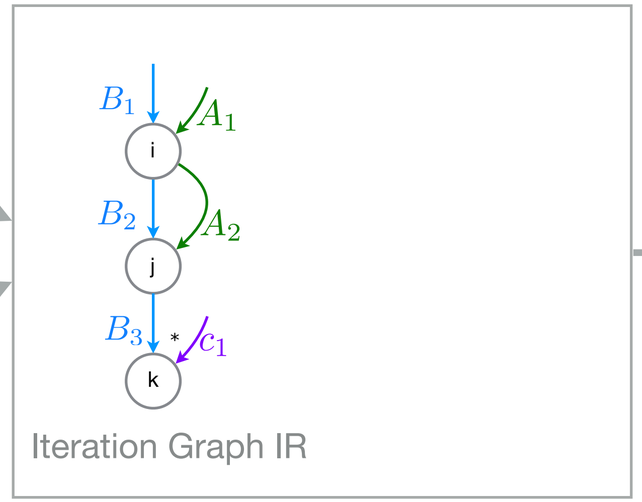
$$A_{ij} = \sum_k B_{ijk} c_k$$

Tensor Expression

$A : (\text{dense}, \text{dense})$
$B : (\text{dense}, \text{sparse}, \text{sparse})$
$c : (\text{sparse})$

Tensor Formats

Tensor Algebra Compiler



C code

```
for (int i = 0; i < m; i++) {

  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        a[pA2] += b[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

The Tensor Algebra Compiler (taco)

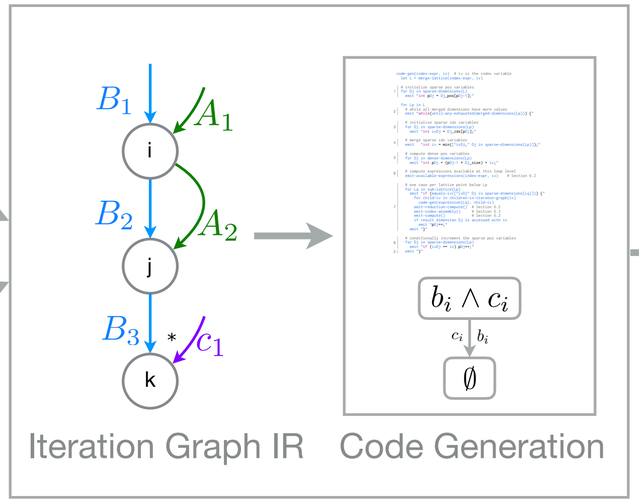
$$A_{ij} = \sum_k B_{ijk} c_k$$

Tensor Expression

$A : (\text{dense}, \text{dense})$
$B : (\text{dense}, \text{sparse}, \text{sparse})$
$c : (\text{sparse})$

Tensor Formats

Tensor Algebra Compiler



C code

```
for (int i = 0; i < m; i++) {

  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        a[pA2] += b[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

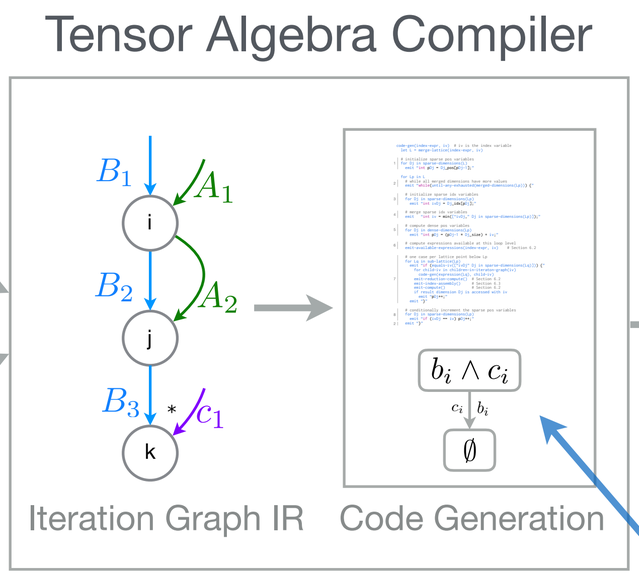
The Tensor Algebra Compiler (taco)

$$A_{ij} = \sum_k B_{ijk} c_k$$

Tensor Expression

$A : (\text{dense}, \text{dense})$
$B : (\text{dense}, \text{sparse}, \text{sparse})$
$c : (\text{sparse})$

Tensor Formats



C code

```
for (int i = 0; i < m; i++) {

  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        a[pA2] += b[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

Merge Lattices

Sparse iteration spaces and Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$

Sparse iteration spaces and Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$

Sparse iteration spaces and Iteration Graphs

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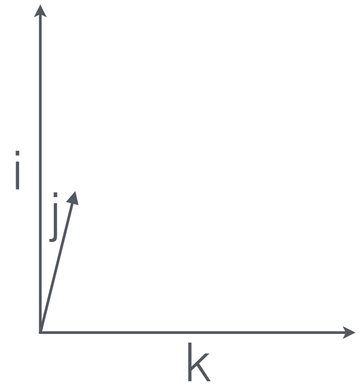
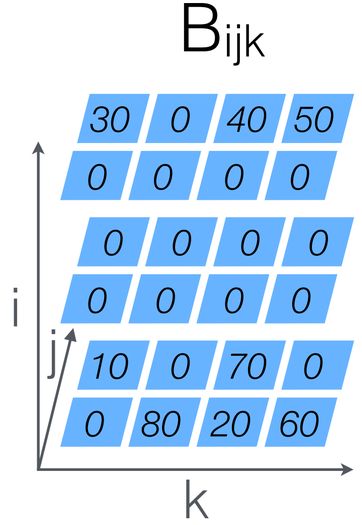


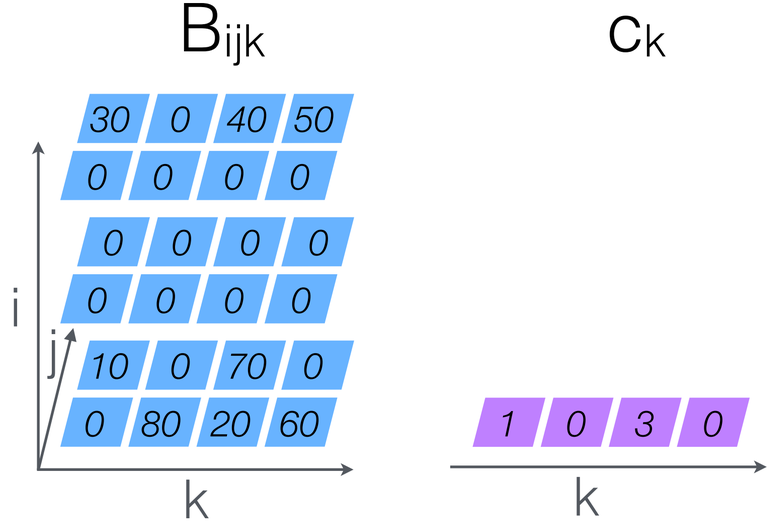
Sparse iteration spaces and Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$



Sparse iteration spaces and Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$



Sparse iteration spaces and Iteration Graphs

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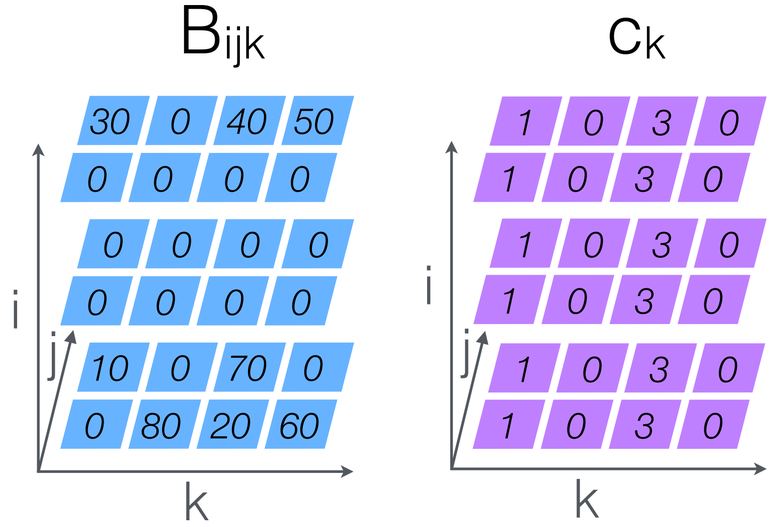
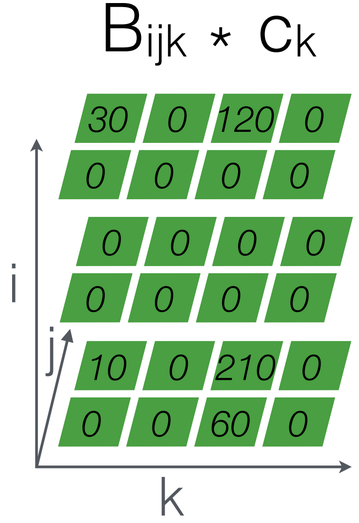


Sparse iteration spaces and Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$

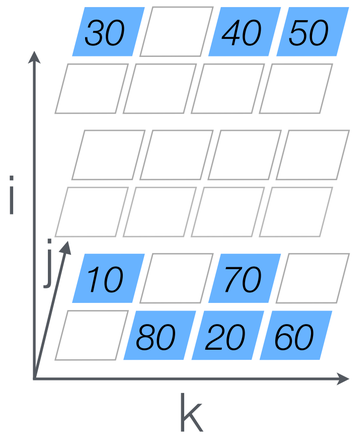


Dense

Sparse iteration spaces and Iteration Graphs

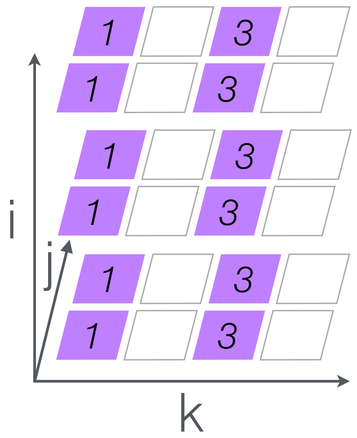
$$A_{ij} = \sum_k B_{ijk} * c_k$$

B_{ijk}

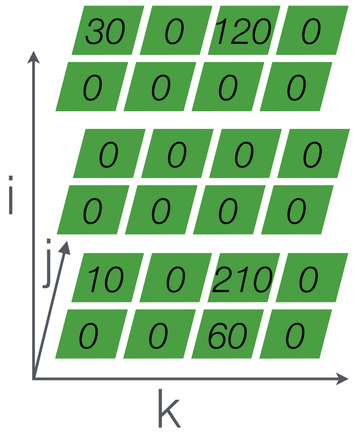


Sparse

c_k



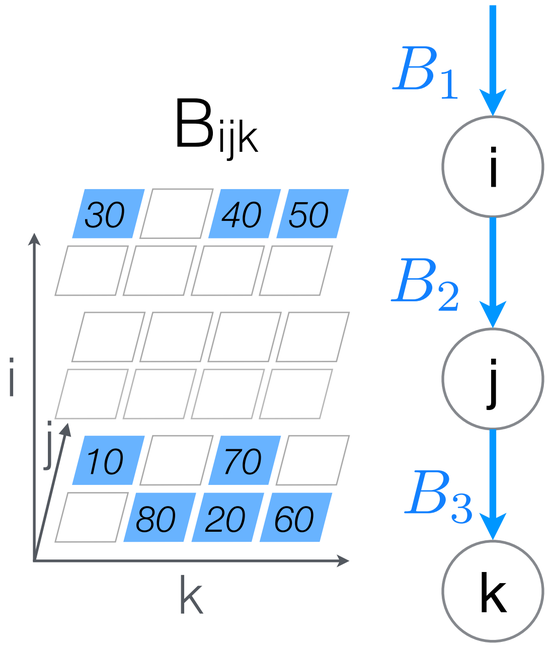
$B_{ijk} * c_k$

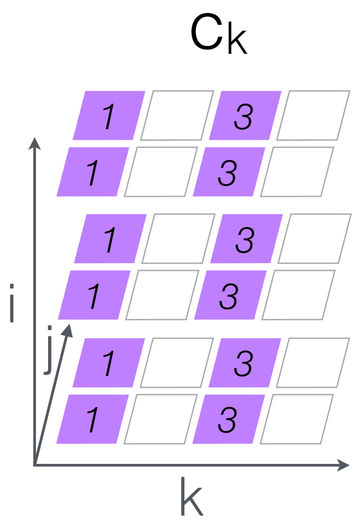


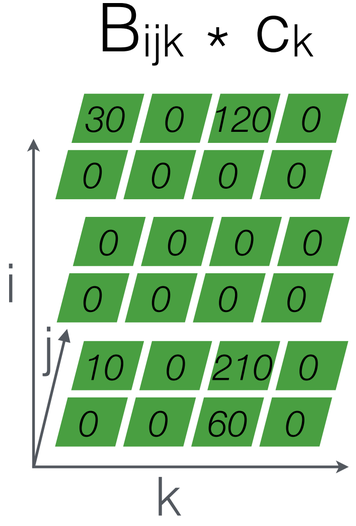
Dense

Sparse iteration spaces and Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$







Sparse

Dense

Sparse iteration spaces and Iteration Graphs

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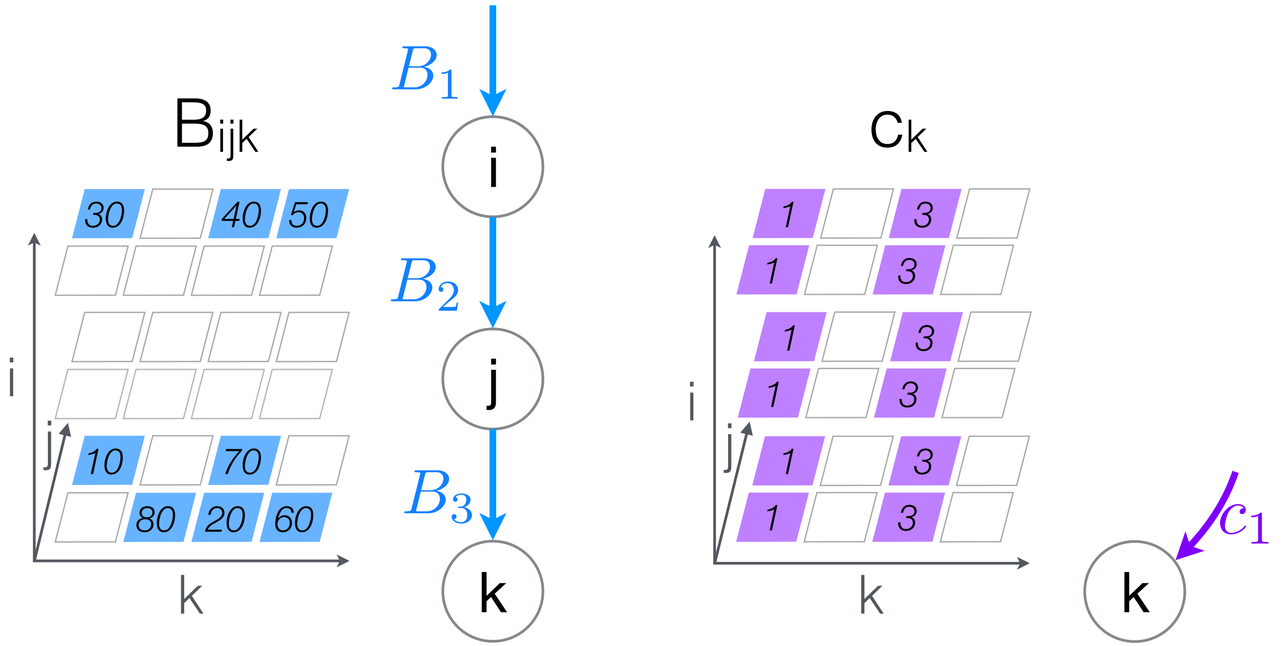

Sparse

Dense

Sparse iteration spaces and Iteration Graphs

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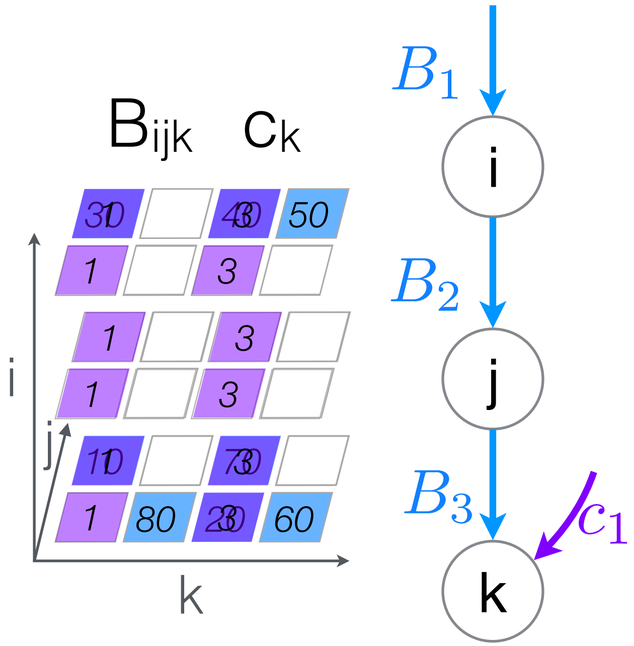
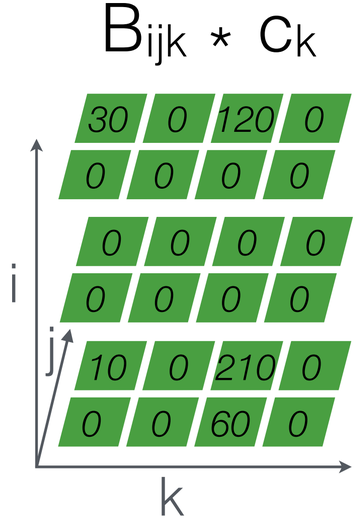


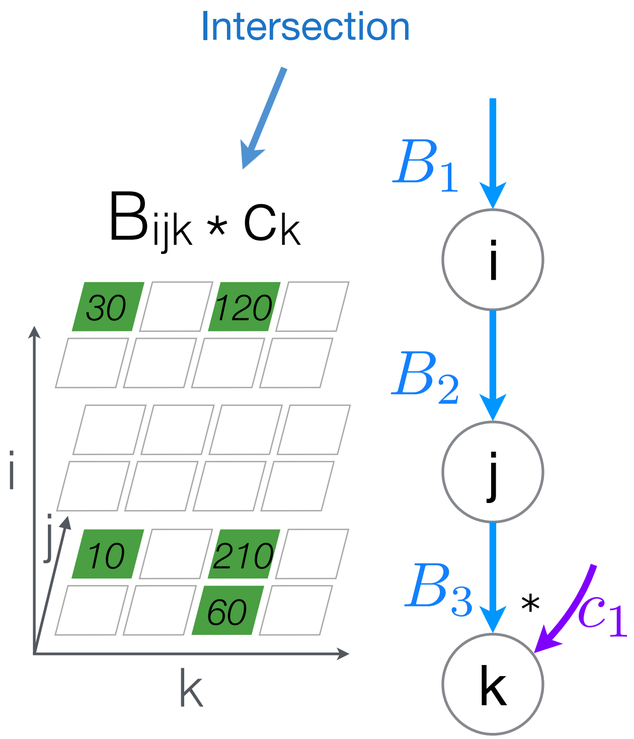
Sparse



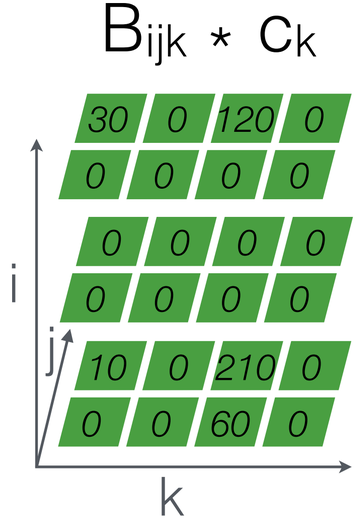
Dense

Sparse iteration spaces and Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$



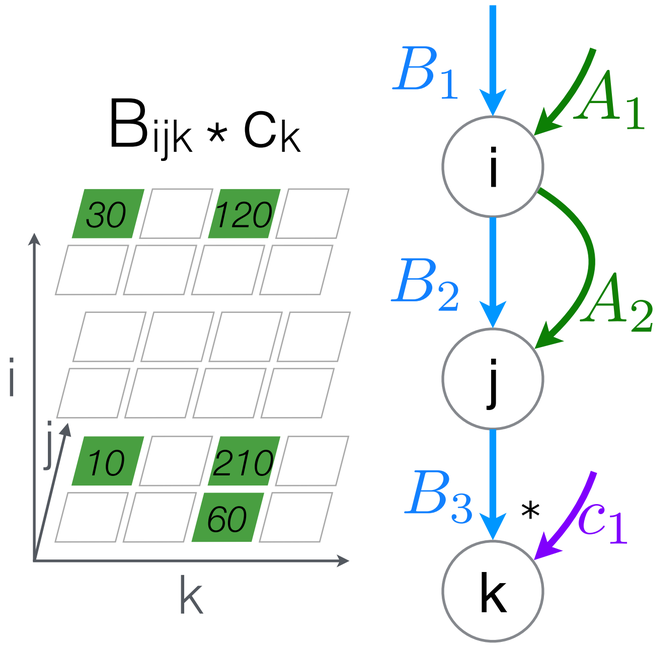
Sparse



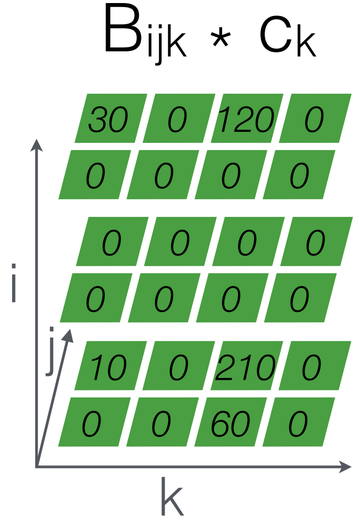
Dense

Sparse iteration spaces and Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$



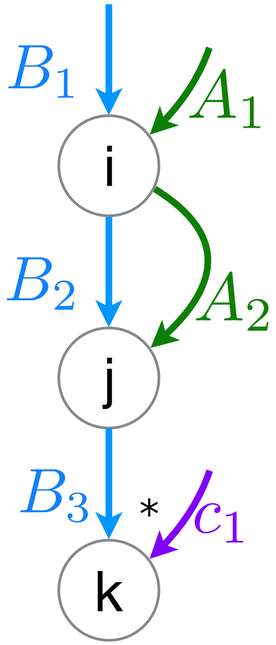
Sparse



Dense

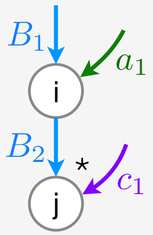
Examples of Iteration Graph

$$A_{ij} = \sum_k B_{ijk} * c_k$$

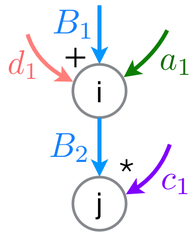


Examples of Iteration Graph

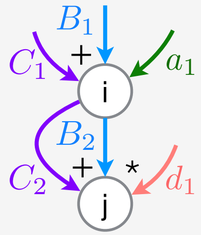
$$a_i = \sum_j B_{ij}\, c_j$$



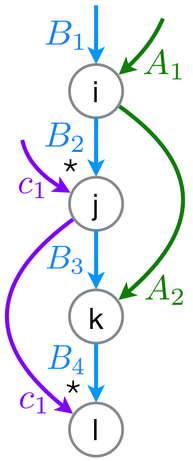
$$a_i = \sum_j \alpha B_{ij}\, c_j + \beta d_i$$



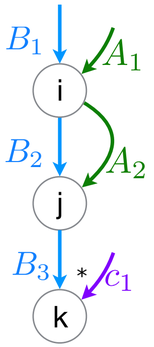
$$a_i = \sum_j (B_{ij} + C_{ij})\, d_j$$



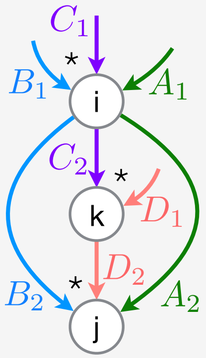
$$a_{ik} = \sum_j \sum_l B_{ijkl}\, c_{jl}$$



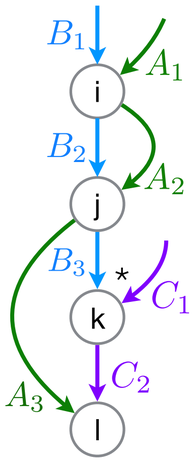
$$A_{ij} = \sum_k B_{ijk} * c_k$$



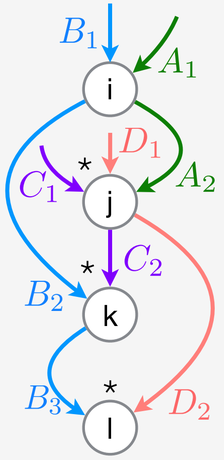
$$A_{ij} = \sum_k B_{ij}(C_{ik} D_{kj})$$



$$A_{ijl} = \sum_k B_{ijk} C_{lk}$$

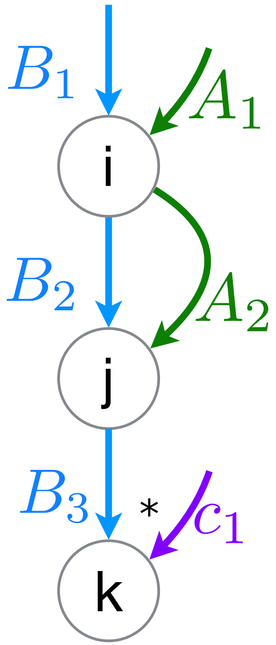


$$A_{ij} = \sum_k \sum_l B_{ikl} C_{kj} D_{lj}$$



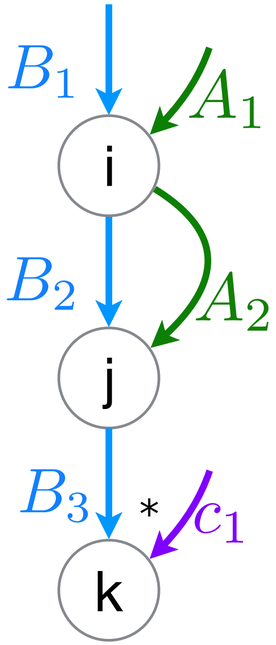
Examples of Iteration Graph

$$A_{ij} = \sum_k B_{ijk} * c_k$$



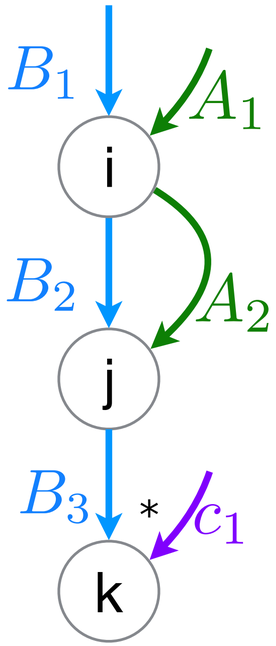
Code Generation from Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$



Code Generation from Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$



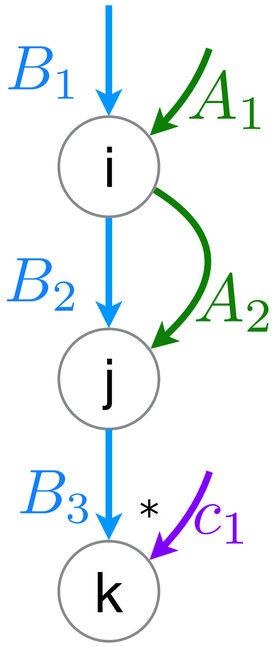
dense dimension

```
for (int i = 0; i < m; i++) {
```

```
}
```

Code Generation from Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$



dense dimension

```
for (int i = 0; i < m; i++) {
```

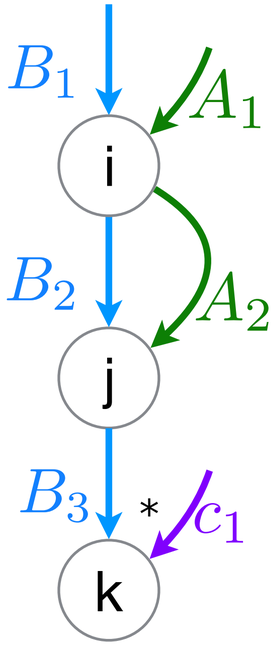
sparse dimension

```
  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;
```

```
  }
}
```

Code Generation from Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$



dense dimension

```
for (int i = 0; i < m; i++) {
```

sparse dimension

```
  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;
```

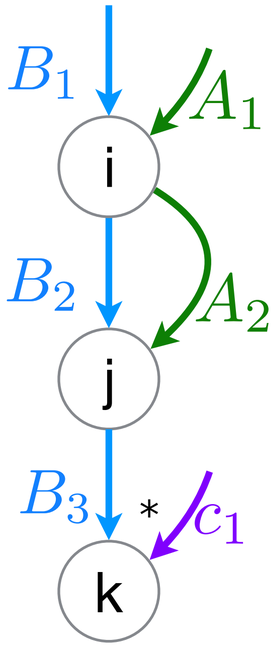
merged dimensions

```
    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {

      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

Code Generation from Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$



dense dimension

sparse dimension

merged dimensions

compute statement

```
for (int i = 0; i < m; i++) {

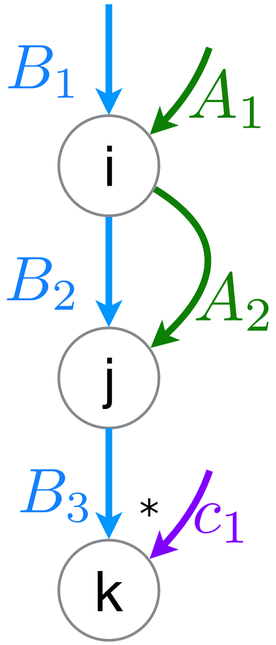
  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        A[pA2] += B[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }

  }
}
```

Code Generation from Iteration Graphs

$$A_{ij} = \sum_k B_{ijk} * c_k$$



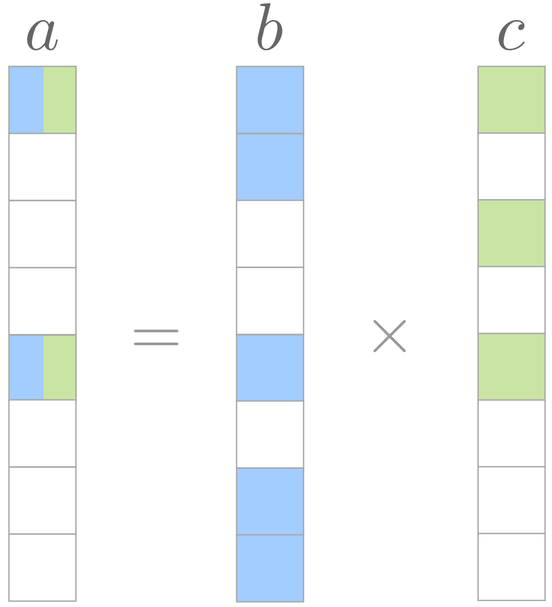

```
for (int i = 0; i < m; i++) {


  for (int pB2 = B2_pos[pB1]; pB2 < B2_pos[pB1 + 1]; pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;


    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        A[pA2] += B[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

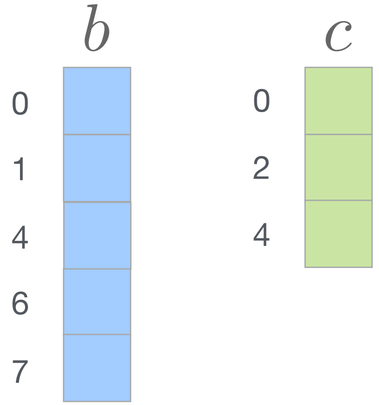
Merge Lattice for a Conjunction

$$a_i = b_i c_i$$



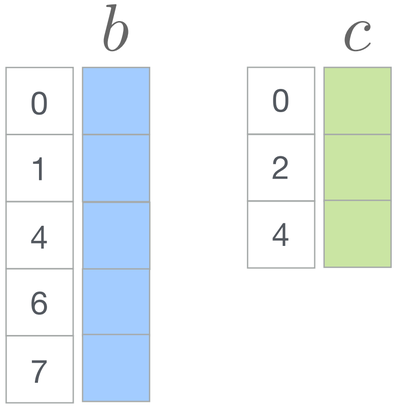
Merge Lattice for a Conjunction

$$a_i = b_i c_i$$



Merge Lattice for a Conjunction

$$a_i = b_i c_i$$



Merge Lattice for a Conjunction

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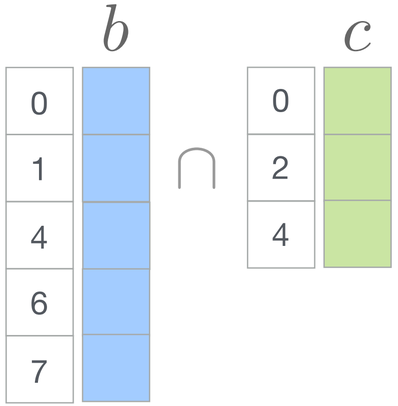

Merge Lattice for a Conjunction

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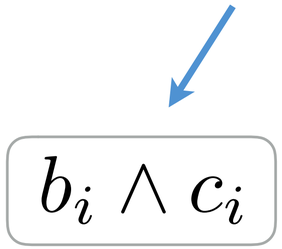
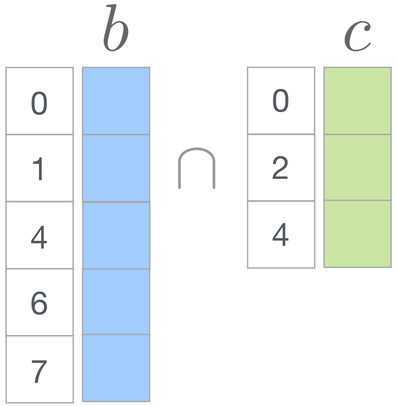
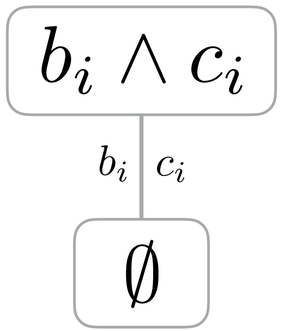
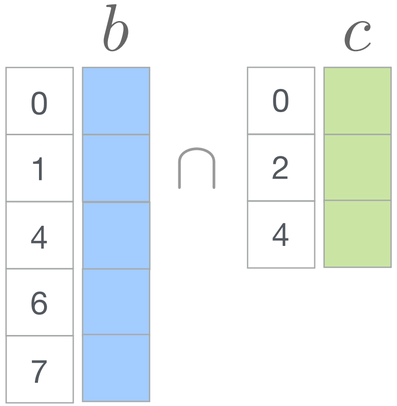




Merge Lattice for a Conjunction

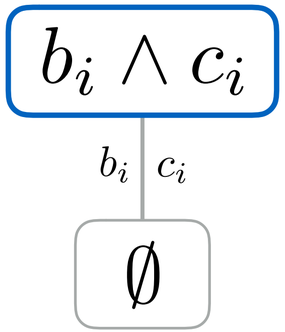
$$a_i = b_i c_i$$

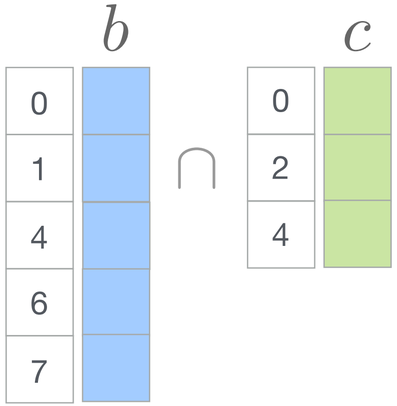




Merge Lattice for a Conjunction

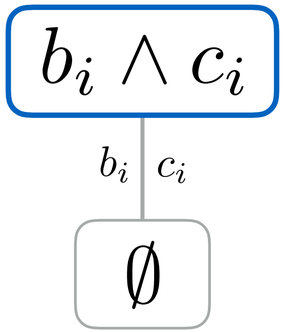
$$a_i = b_i c_i$$





Merge Lattice for a Conjunction

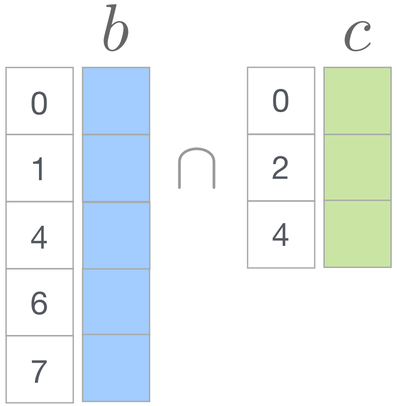
$$a_i = b_i c_i$$




```
int pb1 = b1_pos[0];
int pc1 = c1_pos[0];
while (pb1 < b1_pos[1] && pc1 < c1_pos[1]) {
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int i = min(ib, ic);

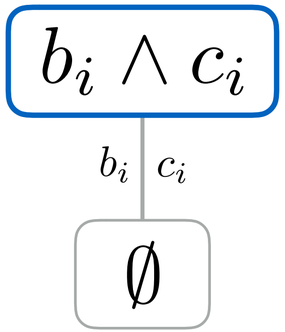


  if (ib == i) pb1++;
  if (ic == i) pc1++;
}
```

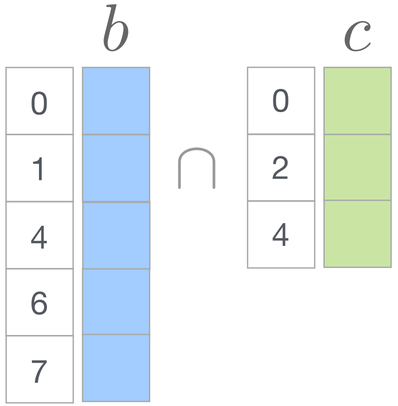


Merge Lattice for a Conjunction

$$a_i = b_i c_i$$

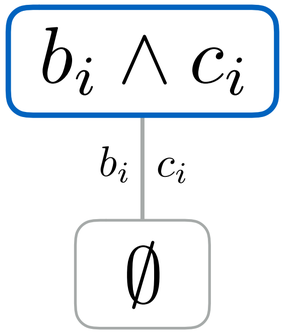



```
int pb1 = b1_pos[0];
int pc1 = c1_pos[0];
while (pb1 < b1_pos[1] && pc1 < c1_pos[1]) {
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int i = min(ib, ic);
  if (ib == i && ic == i) {
    a[i] = b[pb1] * c[pc1];
  }
  if (ib == i) pb1++;
  if (ic == i) pc1++;
}
```



Merge Lattice for a Conjunction

$$a_i = b_i c_i$$

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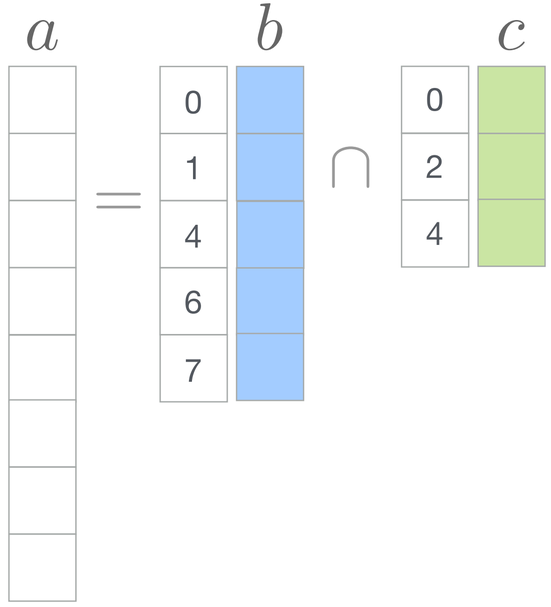
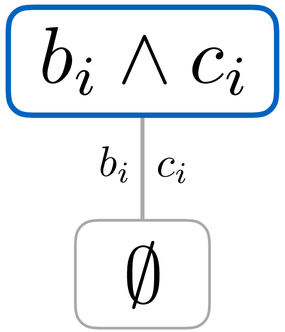
    a[i] = b[pb1] * c[pc1];
  }
  if (ib == i) pb1++;
  if (ic == i) pc1++;
}
```



Merge Lattice for a Conjunction

$$a_i = b_i c_i$$

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  int i = min(ib, ic);
  if (ib == i && ic == i) {
    a[i] = b[pb1] * c[pc1];
  }
  if (ib == i) pb1++;
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}
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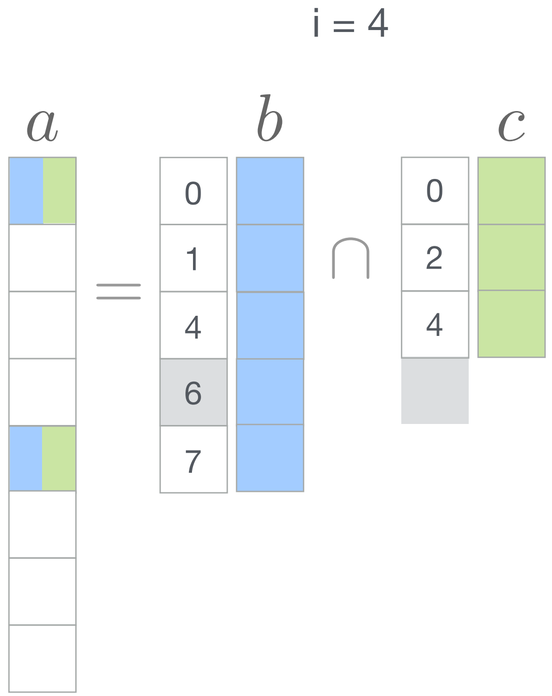
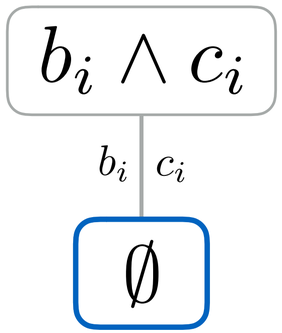


Merge Lattice for a Conjunction

$$a_i = b_i c_i$$

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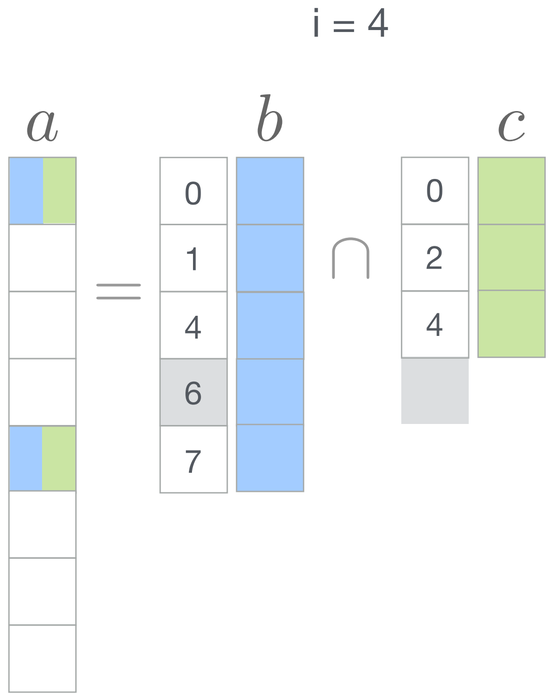
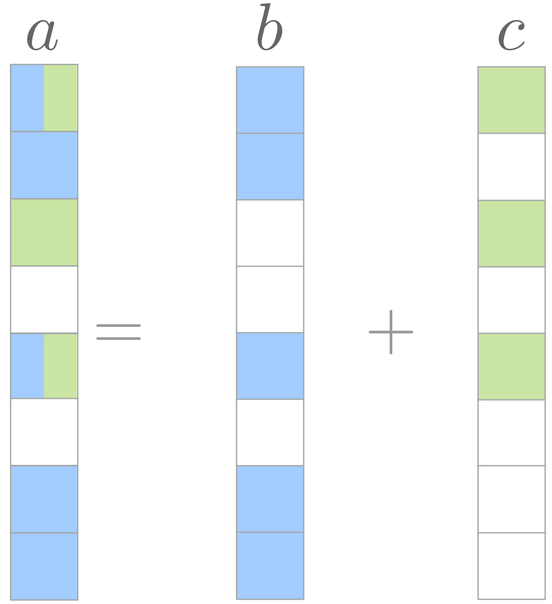
while (pb1 < b1_pos[1] && pc1 < c1_pos[1]) {
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int i = min(ib, ic);
  if (ib == i && ic == i) {
    a[i] = b[pb1] * c[pc1];
  }
  if (ib == i) pb1++;
  if (ic == i) pc1++;
}
```



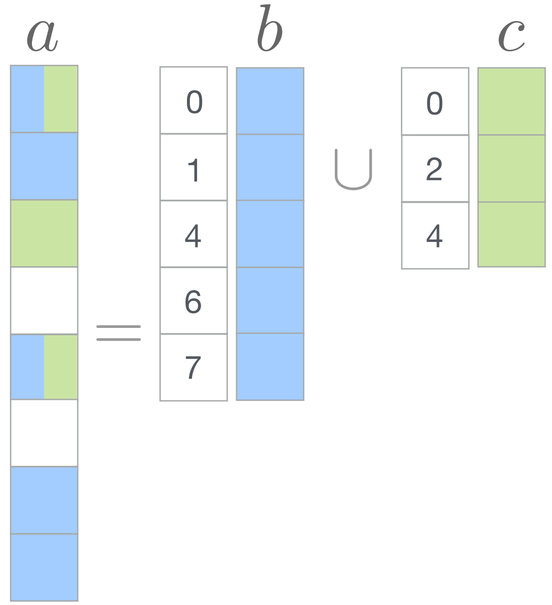
Merge Lattice for a Disjunction

$$a_i = b_i + c_i$$



Merge Lattice for a Disjunction

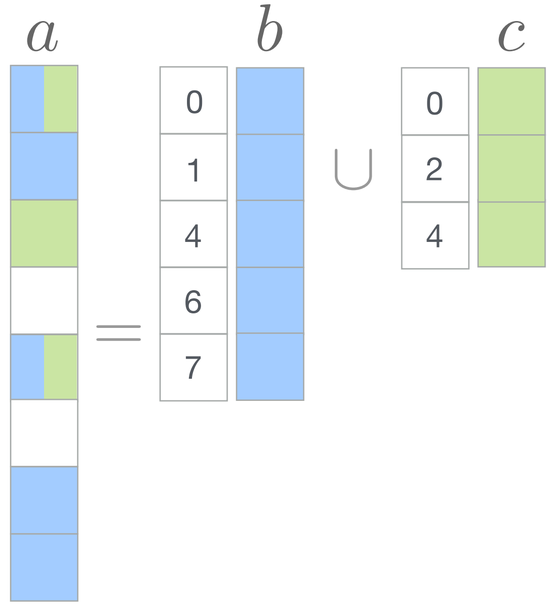
$$a_i = b_i + c_i$$



Merge Lattice for a Disjunction

$$a_i = b_i + c_i$$

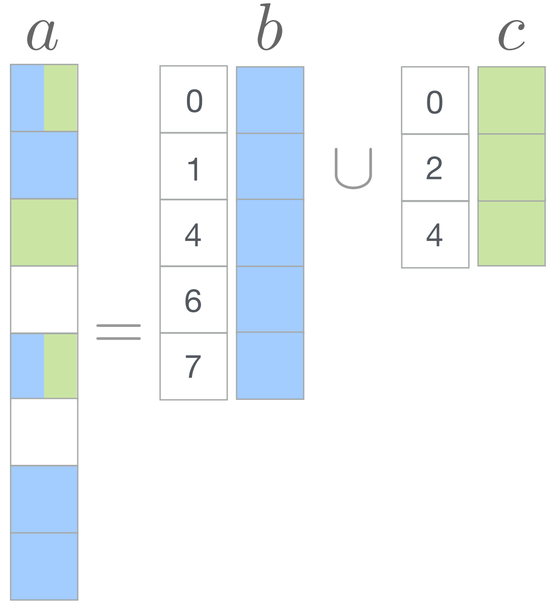
$b_i \vee c_i$



Merge Lattice for a Disjunction

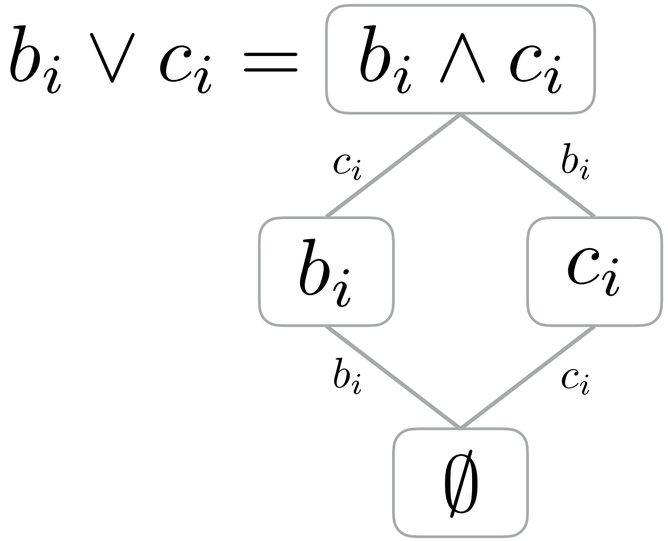
$$a_i = b_i + c_i$$

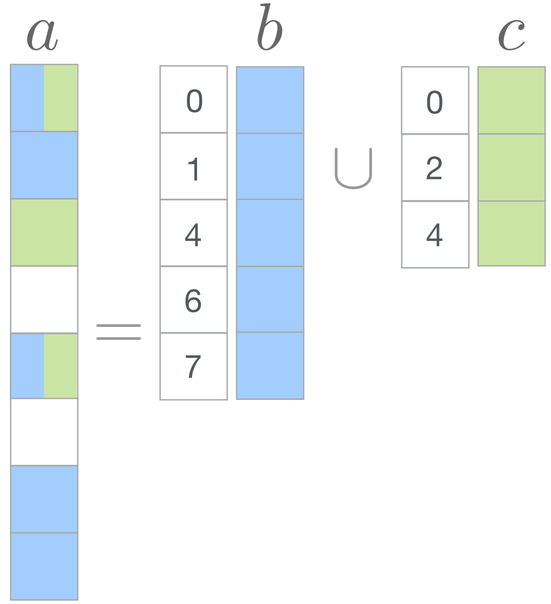
$$b_i \vee c_i = \boxed{b_i \wedge c_i} \vee b_i \vee c_i$$



Merge Lattice for a Disjunction

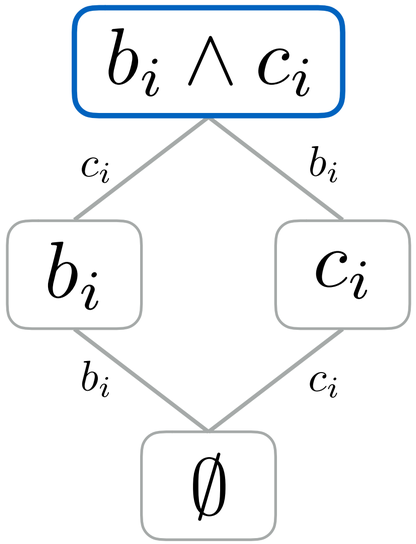
$$a_i = b_i + c_i$$

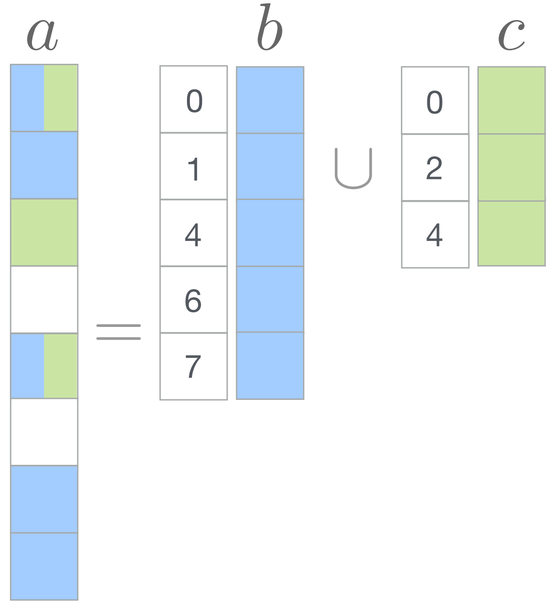




Merge Lattice for a Disjunction

$$a_i = b_i + c_i$$





Merge Lattice for a Disjunction

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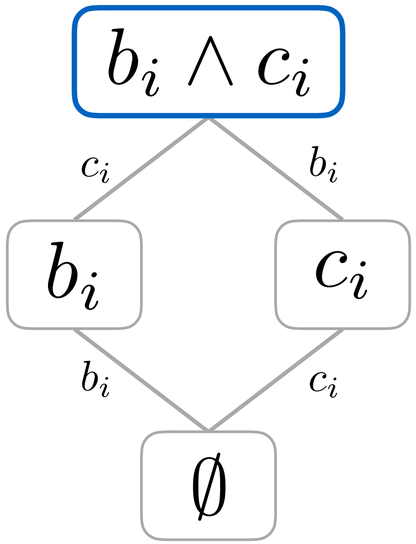



```
int pb1 = b1_pos[0];
int pc1 = c1_pos[0];
while (pb1 < b1_pos[1] && pc1 < c1_pos[1]) {
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int i = min(ib, ic);

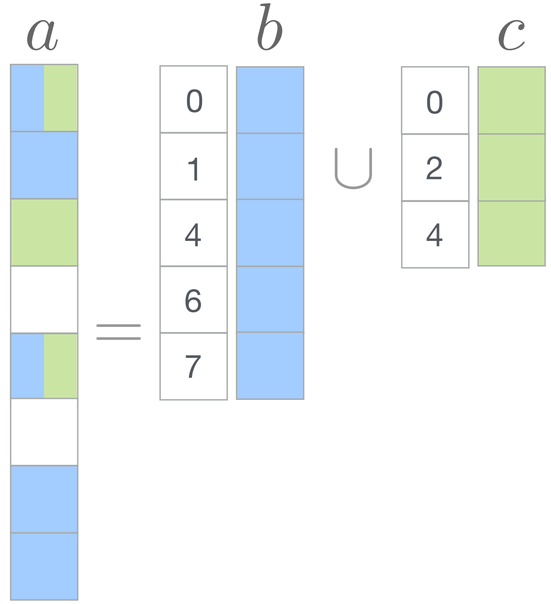








  if (ib == i) pb1++;
  if (ic == i) pc1++;
}
```



Merge Lattice for a Disjunction

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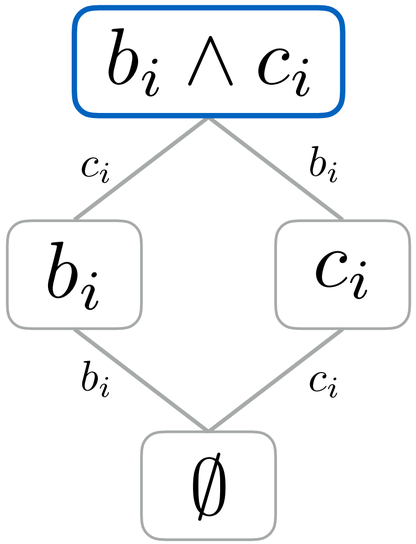


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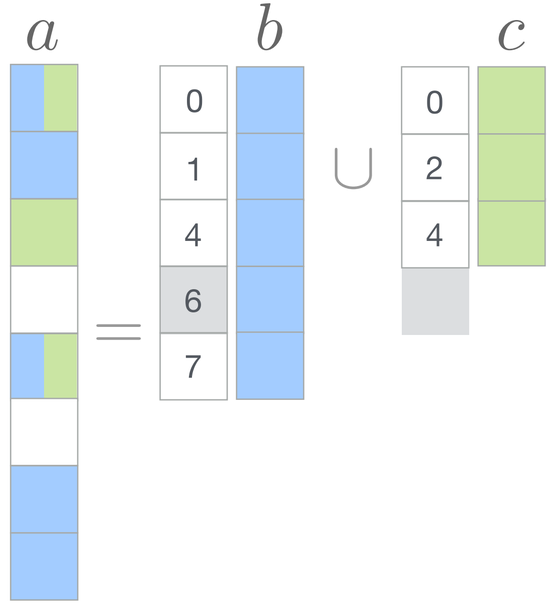
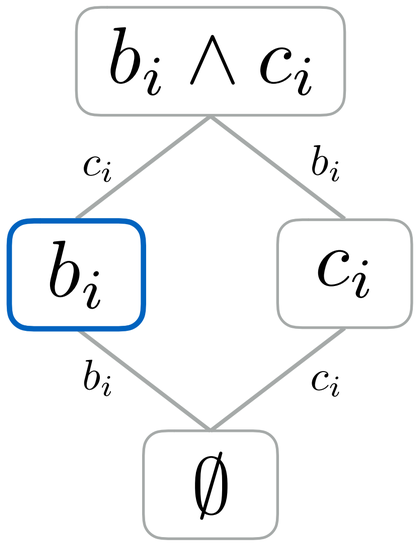
  if (ib == i) pb1++;
  if (ic == i) pc1++;
}
```



Merge Lattice for a Disjunction

$$a_i = b_i + c_i$$

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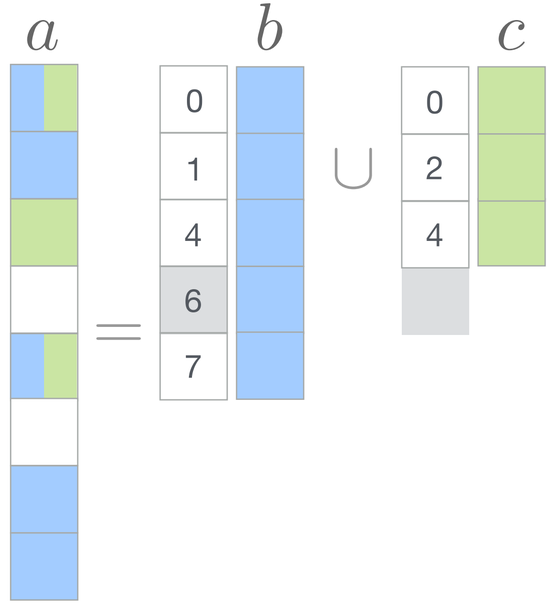
  int ic = c1_idx[pc1];
  int i = min(ib, ic);

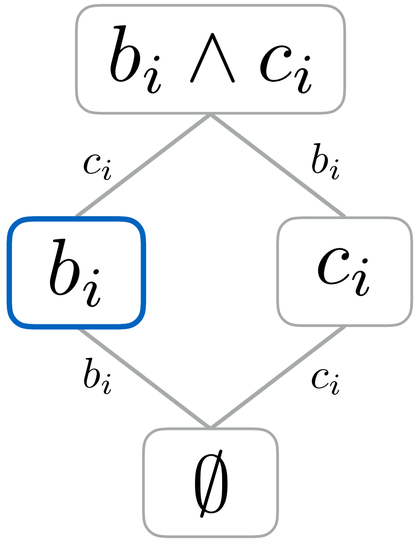



  if (ib == i) pb1++;
  if (ic == i) pc1++;
}
```



Merge Lattice for a Disjunction

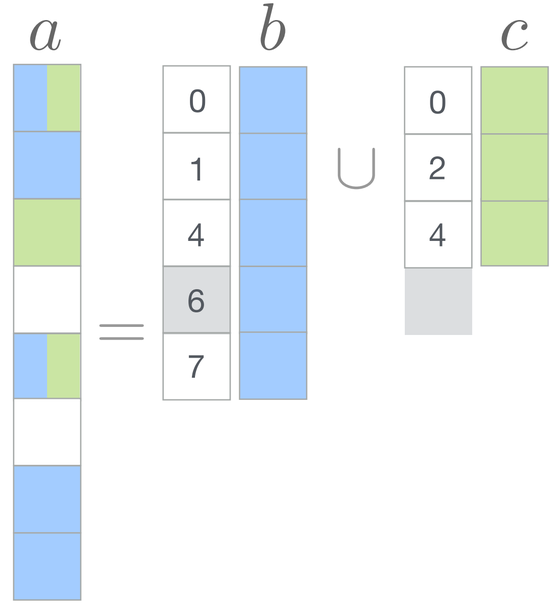
$$a_i = b_i + c_i$$




```
int pb1 = b1_pos[0];
int pc1 = c1_pos[0];
while (pb1 < b1_pos[1] && pc1 < c1_pos[1]) {
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int i = min(ib, ic);
```

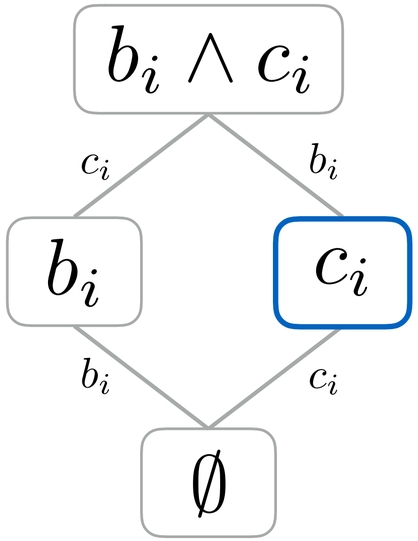
```
  if (ib == i) pb1++;
  if (ic == i) pc1++;
}
```

```
while (pb1 < b1_pos[1]) {
  int i = b1_idx[pb1];
  a[i] = b[pb1++];
}
```



Merge Lattice for a Disjunction

$$a_i = b_i + c_i$$

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while (pb1 < b1_pos[1] && pc1 < c1_pos[1]) {
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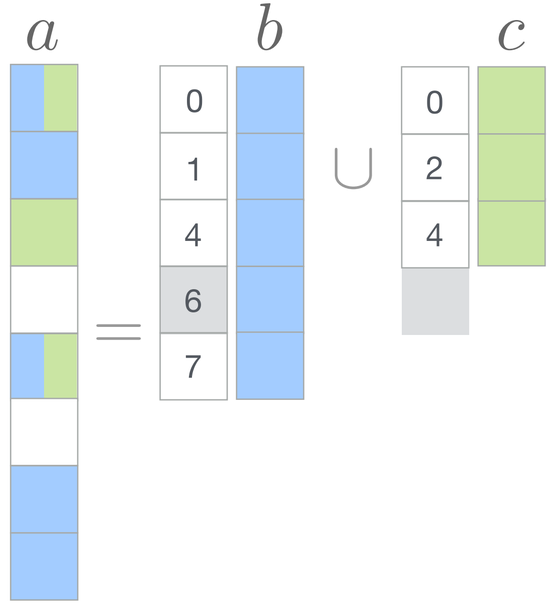
  int ic = c1_idx[pc1];
  int i = min(ib, ic);

  if (ib == i) pb1++;
  if (ic == i) pc1++;
}

while (pb1 < b1_pos[1]) {
  int i = b1_idx[pb1];
  a[i] = b[pb1++];
}

while (pc1 < c1_pos[1]) {
  int i = c1_idx[pc1];
  a[i] = c[pc1++];
}
```



Merge Lattice for a Disjunction

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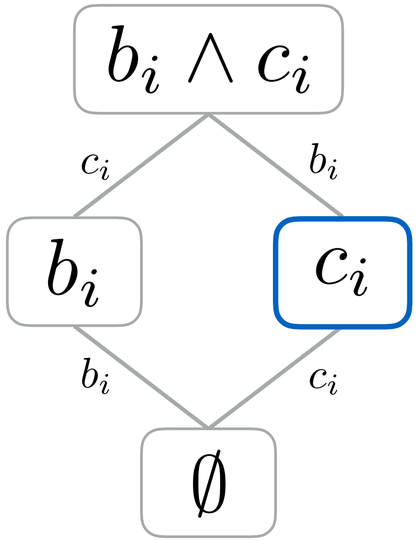


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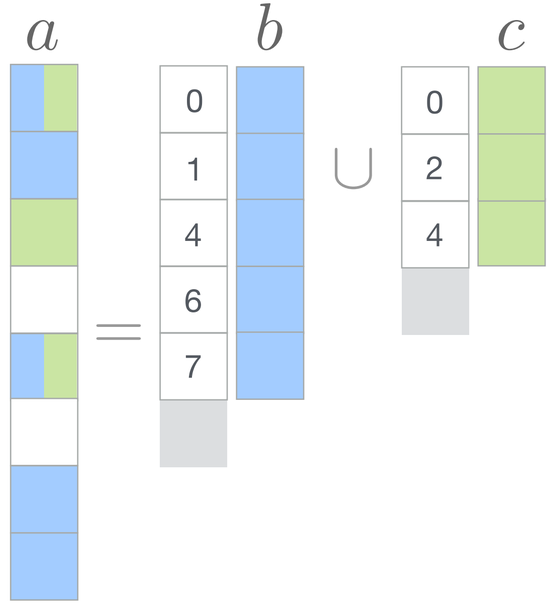
  int ic = c1_idx[pc1];
  int i = min(ib, ic);
  if (ib == i) pb1++;
  if (ic == i) pc1++;
}

while (pb1 < b1_pos[1]) {
  int i = b1_idx[pb1];
  a[i] = b[pb1++];
}

while (pc1 < c1_pos[1]) {
  int i = c1_idx[pc1];
  a[i] = c[pc1++];
}
```



Merge Lattice for a Disjunction

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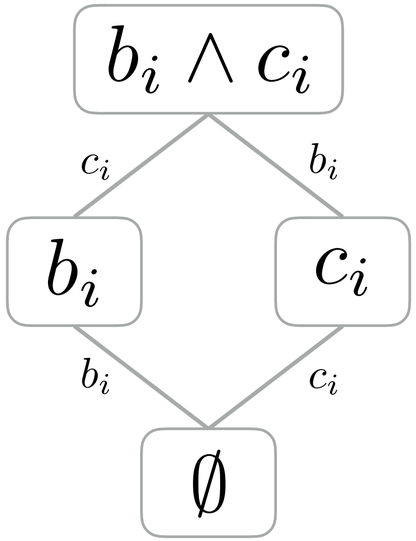


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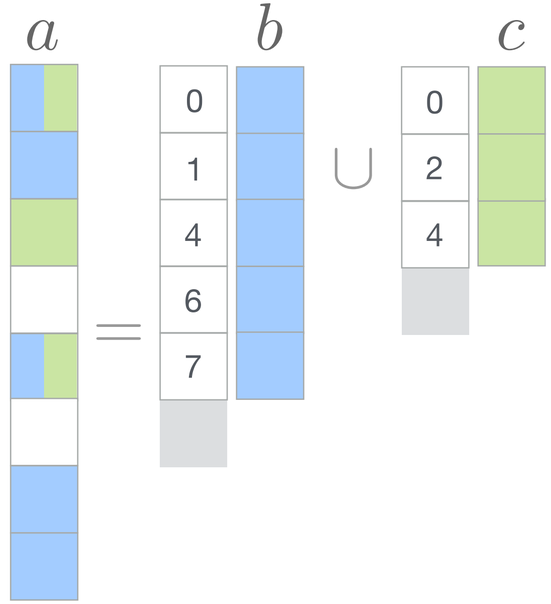
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int i = min(ib, ic);

  if (ib == i) pb1++;
  if (ic == i) pc1++;
}

while (pb1 < b1_pos[1]) {
  int i = b1_idx[pb1];
  a[i] = b[pb1++];
}

while (pc1 < c1_pos[1]) {
  int i = c1_idx[pc1];
  a[i] = c[pc1++];
}
```



Merge Lattice for a Disjunction

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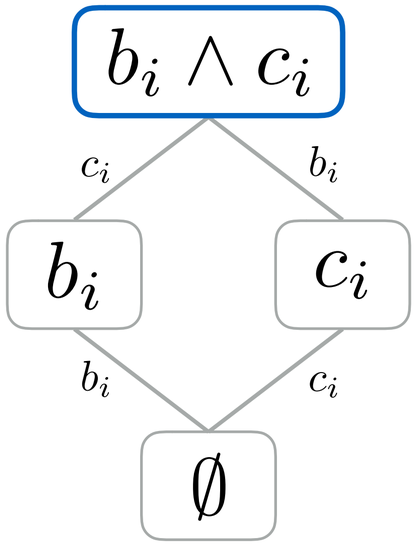


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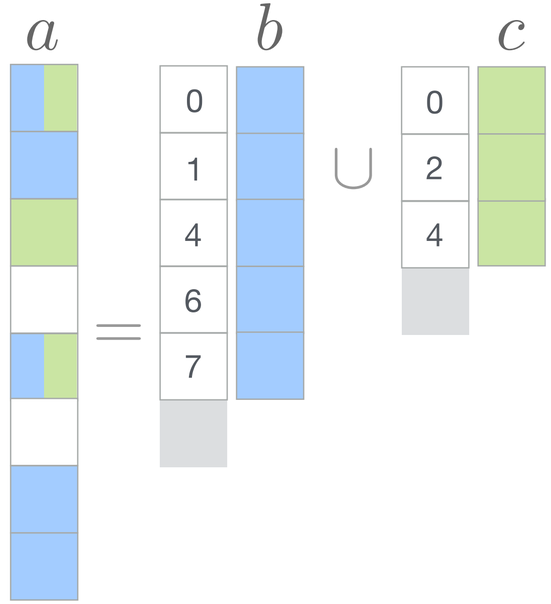
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int i = min(ib, ic);
```

```
  if (ib == i) pb1++;
  if (ic == i) pc1++;
}

while (pb1 < b1_pos[1]) {
  int i = b1_idx[pb1];
  a[i] = b[pb1++];
}

while (pc1 < c1_pos[1]) {
  int i = c1_idx[pc1];
  a[i] = c[pc1++];
}
```



Merge Lattice for a Disjunction

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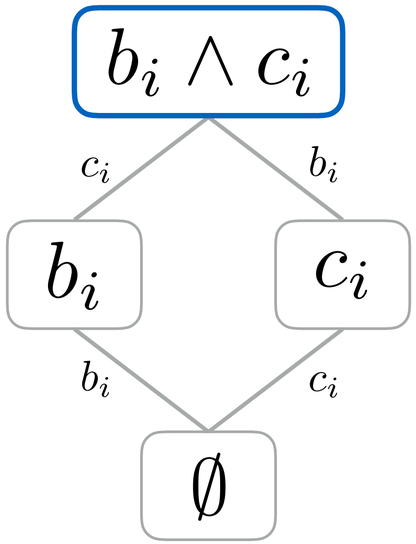


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  int ic = c1_idx[pc1];
  int i = min(ib, ic);
  if (ib == i && ic == i) {
    a[i] = b[pb1] + c[pc1];
  }
  if (ib == i) pb1++;
  if (ic == i) pc1++;
}

while (pb1 < b1_pos[1]) {
  int i = b1_idx[pb1];
  a[i] = b[pb1++];
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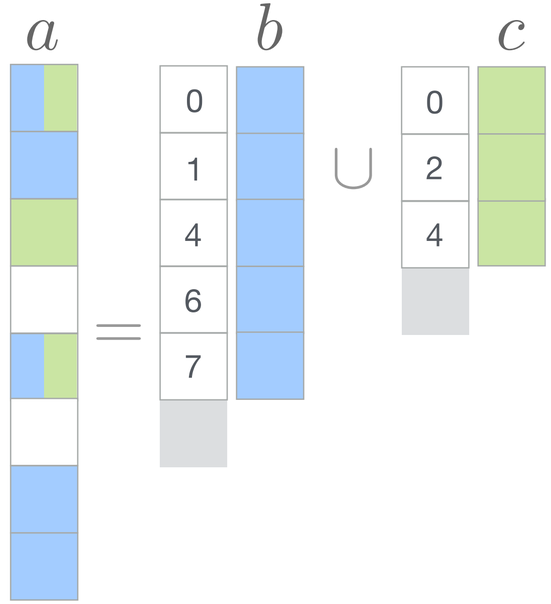
  int i = c1_idx[pc1];
  a[i] = c[pc1++];
}
```



Merge Lattice for a Disjunction

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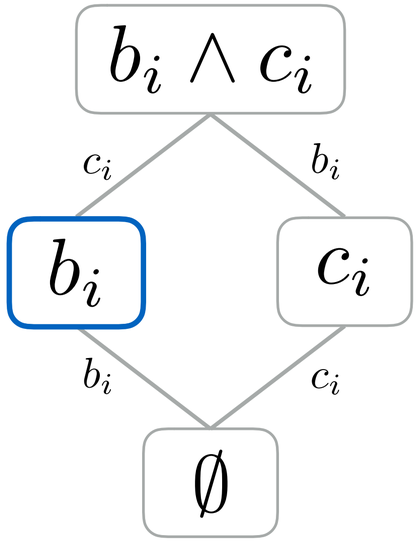


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int pc1 = c1_pos[0];
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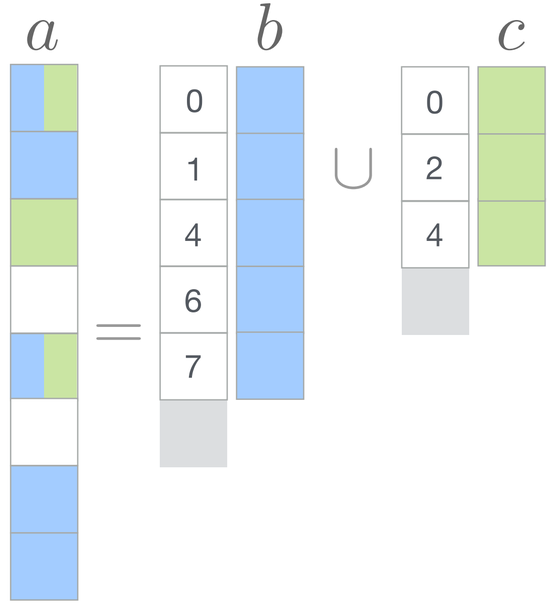
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int i = min(ib, ic);
  if (ib == i && ic == i) {
    a[i] = b[pb1] + c[pc1];
  }
  else if (ib == i) {
    a[i] = b[pb1];
  }


  if (ib == i) pb1++;
  if (ic == i) pc1++;
}

while (pb1 < b1_pos[1]) {
  int i = b1_idx[pb1];
  a[i] = b[pb1++];
}

while (pc1 < c1_pos[1]) {
  int i = c1_idx[pc1];
  a[i] = c[pc1++];
}
```



Merge Lattice for a Disjunction

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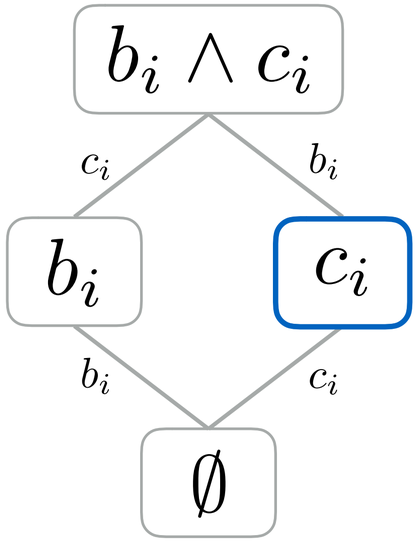


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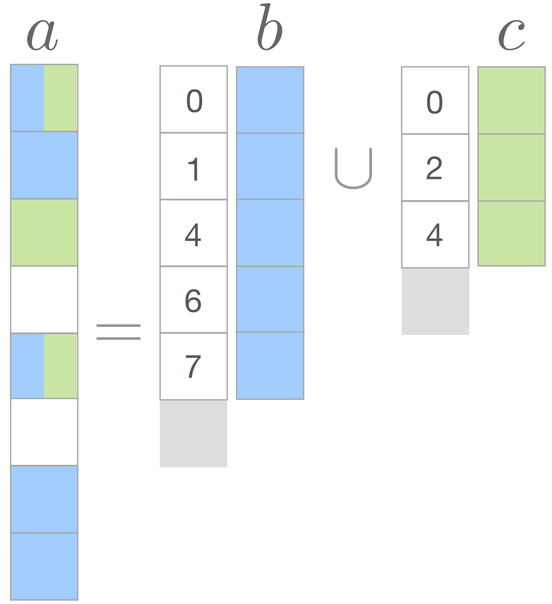
int pb1 = b1_pos[0];
int pc1 = c1_pos[0];
while (pb1 < b1_pos[1] && pc1 < c1_pos[1]) {
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int i = min(ib, ic);
  if (ib == i && ic == i) {
    a[i] = b[pb1] + c[pc1];
  }
  else if (ib == i) {
    a[i] = b[pb1];
  }
  else {
    a[i] = c[pc1];
  }
  if (ib == i) pb1++;
  if (ic == i) pc1++;
}

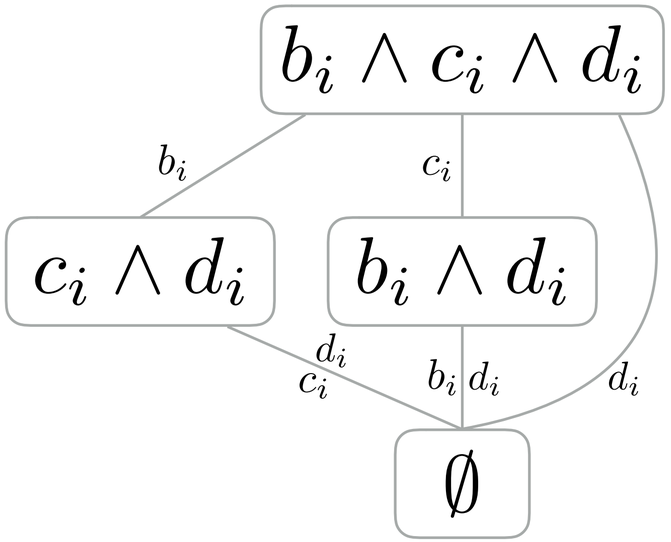
while (pb1 < b1_pos[1]) {
  int i = b1_idx[pb1];
  a[i] = b[pb1++];
}

while (pc1 < c1_pos[1]) {
  int i = c1_idx[pc1];
  a[i] = c[pc1++];
}
```



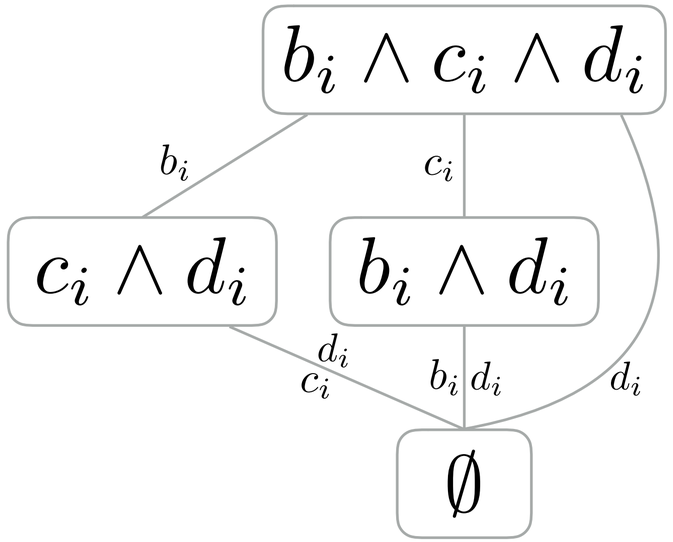
Merge Lattice for a Compound Expression

$$a_i = (b_i + c_i)d_i$$



Merge Lattice for a Compound Expression

$$a_i = (b_i + c_i)d_i$$



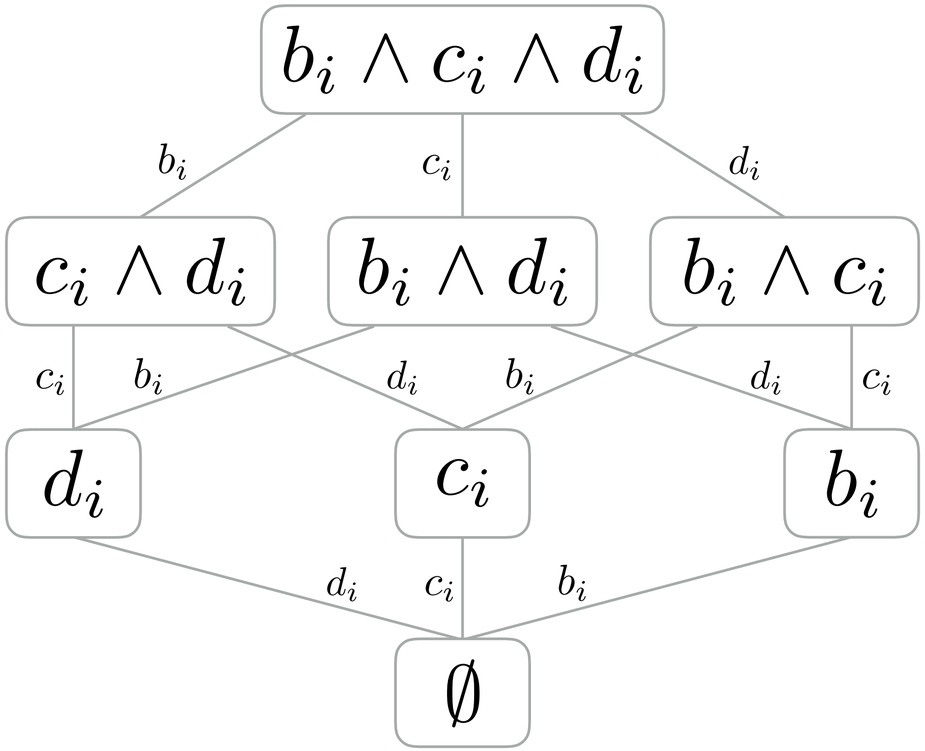

```
int pb1 = b1_pos[0];
int pc1 = c1_pos[0];
int pd1 = d1_pos[0];
while (pb1 < b1_pos[1] && pc1 < c1_pos[1] && pd1 < d1_pos[1]) {
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int id = d1_idx[pd1];
  int i = min(ib, ic, id);
  if (ib == i && ic == i && id == i) {
    a[i] = (b[pb1] + c[pc1]) * d[pd1];
  }
  else if (ib == i && id == i) {
    a[i] = b[pb1] * d[pd1];
  }
  else if (ic == i && id == i) {
    a[i] = c[pc1] * d[pd1];
  }
  if (ib == i) pb1++;
  if (ic == i) pc1++;
  if (id == i) pd1++;
}

while (pc1 < c1_pos[1] && pd1 < d1_pos[1]) {
  int ic0 = c1_idx[pc1];
  int id1 = d1_idx[pd1];
  int i1 = min(ic0, id1);
  if (ic0 == i1 && id1 == i1) {
    a[i1] = c[pc1] * d[pd1];
  }
  if (ic0 == i1) pc1++;
  if (id1 == i1) pd1++;
}

while (pb1 < b1_pos[1] && pd1 < d1_pos[1]) {
  int ib0 = b1_idx[pb1];
  int id0 = d1_idx[pd1];
  int i0 = min(ib0, id0);
  if (ib0 == i0 && id0 == i0) {
    a[i0] = b[pb1] * d[pd1];
  }
  if (ib0 == i0) pb1++;
  if (id0 == i0) pd1++;
}
```

Merge Lattice for a Compound Expression

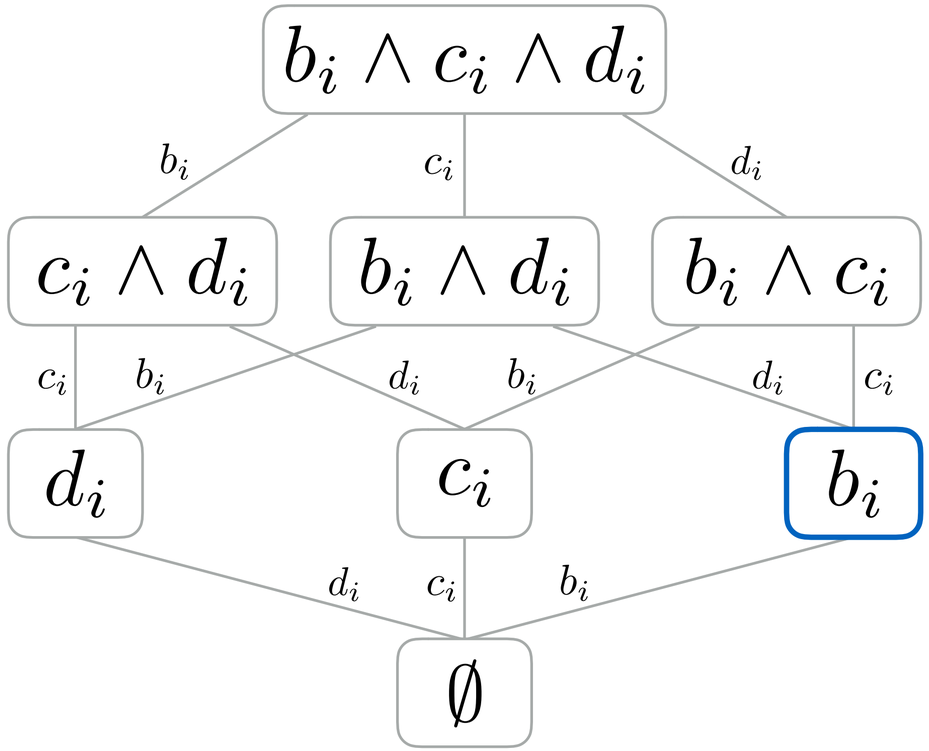
$$a_i = b_i + c_i + d_i$$



Merge Lattice for a Compound Expression

$$a_i = b_i + c_i + d_i$$

dense?




```
int pb1 = b1_pos[0];
int pc1 = c1_pos[0];
int pd1 = d1_pos[0];
while (pb1 < b1_pos[1] && pc1 < c1_pos[1] && pd1 < d1_pos[1]) {
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int id = d1_idx[pd1];
  int i = min(ib, ic, id);
  if (ib == i && ic == i && id == i) {
    a[i] = b[pb1] + c[pc1] + d[pd1];
  }
  else if (ib == i && id == i) {
    a[i] = b[pb1] + d[pd1];
  }
  else if (ic == i && id == i) {
    a[i] = c[pc1] + d[pd1];
  }
  else if (ib == i && ic == i) {
    a[i] = b[pb1] + c[pc1];
  }
  else if (ib == i) {
    a[i] = b[pb1];
  }
  else if (ic == i) {
    a[i] = c[pc1];
  }
  else {
    a[i] = d[pd1];
  }
  if (ib == i) pb1++;
  if (ic == i) pc1++;
  if (id == i) pd1++;
}
```

```
while (pc1 < c1_pos[1] && pd1 < d1_pos[1]) {
  int ic0 = c1_idx[pc1];
  int id1 = d1_idx[pd1];
  int i1 = min(ic0, id1);
  if (ic0 == i1 && id1 == i1) {
    a[i1] = c[pc1] + d[pd1];
  }
  else if (ic0 == i1) {
    a[i1] = c[pc1];
  }
  else {
    a[i1] = d[pd1];
  }
  if (ic0 == i1) pc1++;
  if (id1 == i1) pd1++;
}

while (pb1 < b1_pos[1] && pd1 < d1_pos[1]) {
  int ib0 = b1_idx[pb1];
  int id0 = d1_idx[pd1];
  int i0 = min(ib0, id0);
  if (ib0 == i0 && id0 == i0) {
    a[i0] = b[pb1] + d[pd1];
  }
  else if (ib0 == i0) {
    a[i0] = b[pb1];
  }
  else {
    a[i0] = d[pd1];
  }
  if (ib0 == i0) pb1++;
  if (id0 == i0) pd1++;
}

while (pb1 < b1_pos[1] && pc1 < c1_pos[1]) {
  int ib1 = b1_idx[pb1];
  int ic1 = c1_idx[pc1];
  int i2 = min(ib1, ic1);
  if (ib1 == i2 && ic1 == i2) {
    a[i2] = b[pb1] + c[pc1];
  }
  else if (ib1 == i2) {
    a[i2] = b[pb1];
  }
  else {
    a[i2] = c[pc1];
  }
  if (ib1 == i2) pb1++;
  if (ic1 == i2) pc1++;
}

while (pd1 < d1_pos[1]) {
  int id2 = d1_idx[pd1];
  a[id2] = d[pd1];
  pd1++;
}

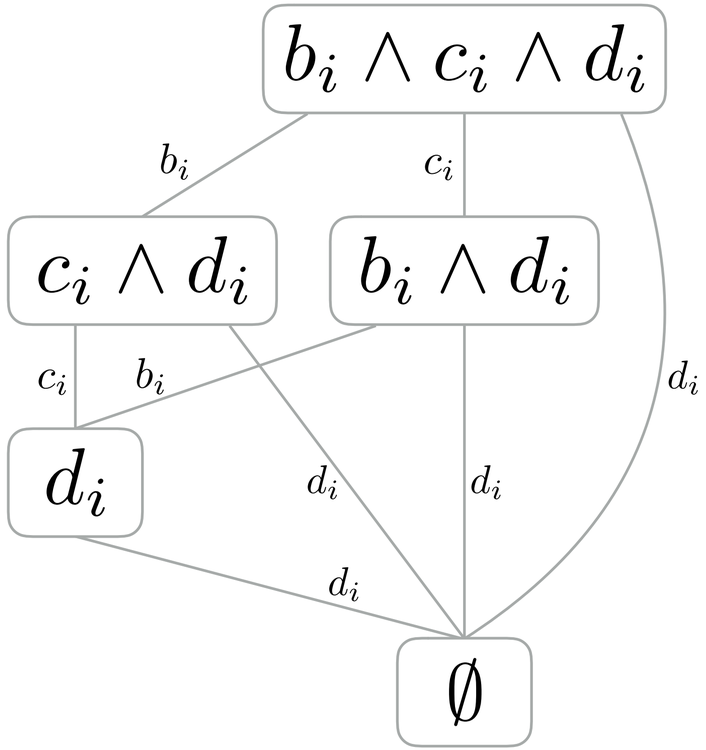
while (pc1 < c1_pos[1]) {
  int ic2 = c1_idx[pc1];
  a[ic2] = c[pc1];
  pc1++;
}

while (pb1 < b1_pos[1]) {
  int ib2 = b1_idx[pb1];
  a[ib2] = b[pb1];
  pb1++;
}
```

Merge Lattice for a Compound Expression

$$a_i = b_i + c_i + d_i$$

dense



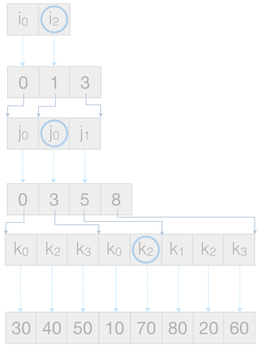

```
int pb1 = b1_pos[0];
int pc1 = c1_pos[0];
int id = 0;
while (pb1 < b1_pos[1] && pc1 < c1_pos[1]) {
  int ib = b1_idx[pb1];
  int ic = c1_idx[pc1];
  int pd1 = id;
  int pa1 = id;
  if (ib == id && ic == id) {
    a[pa1] = b[pb1] + c[pc1] + d[pd1];
  }
  else if (ib == id) {
    a[pa1] = b[pb1] + d[pd1];
  }
  else if (ic == id) {
    a[pa1] = c[pc1] + d[pd1];
  }
  else {
    a[pa1] = d[pd1];
  }
  if (ib == id) pb1++;
  if (ic == id) pc1++;
  id++;
}
```

```
while (pc1 < c1_pos[1]) {
  int ic0 = c1_idx[pc1];
  int pd11 = id;
  int pa11 = id;
  if (ic0 == id) {
    a[pa11] = c[pc1] + d[pd11];
  }
  else {
    a[pa11] = d[pd11];
  }
  if (ic0 == id) pc1++;
  id++;
}
```

```
while (pb1 < b1_pos[1]) {
  int ib0 = b1_idx[pb1];
  int pd10 = id;
  int pa10 = id;
  if (ib0 == id) {
    a[pa10] = b[pb1] + d[pd10];
  }
  else {
    a[pa10] = d[pd10];
  }
  if (ib0 == id) pb1++;
  id++;
}
```

```
while (id < d1_dimension) {
  int pd12 = id;
  int pa12 = id;
  a[pa12] = d[pd12];
  id++;
}
```

Sparse code and data

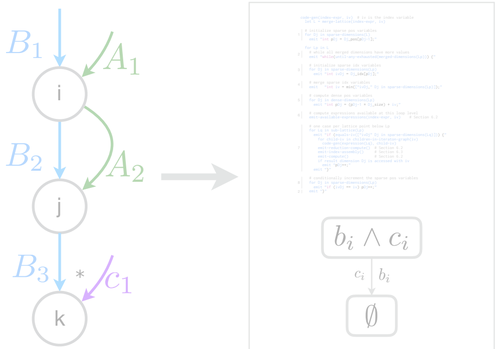



```
for (int pB1 = B1_pos[0]; pB1 < B1_pos[1]; pB1++) {
  int i = B1_idx[pB1];

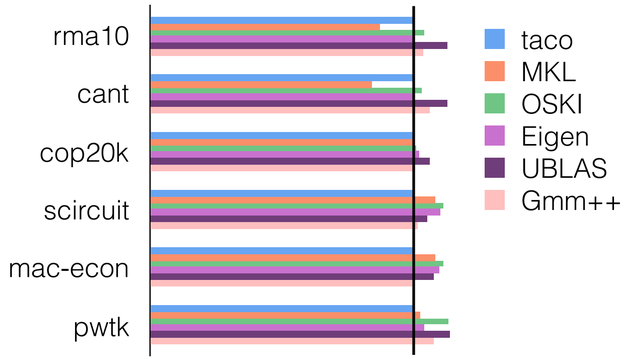
  for (int pB2 = B2_pos[pB1];
           pB2 < B2_pos[pB1 + 1];
           pB2++) {
    int j = B2_idx[pB2];
    int pA2 = (i * n) + j;

    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2 + 1] && pc1 < c1_pos[1]) {
      int kB = B3_idx[pB3];
      int kc = c1_idx[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        a[pA2] += b[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    }
  }
}
```

Intermediate Representation and Code Generation



Evaluation



The Tensor Algebra Compiler is fast and general

$$y = \alpha A^T x + \beta z \quad a = Bc$$

$$a = B^T c \quad A = B + C + D \quad y = b - Ax$$

$$A = B \quad a = Bc + a \quad A_{ijk} = \textstyle\sum_l B_{ijl} C_{kl} \quad A = B \circ (CD)$$

$$A = B \odot C \quad A_{ijk} = B_{ijk} + C_{ijk} \quad A_{ij} \textstyle\sum_k B_{ijk} c_k$$

$$A = B^T \quad \alpha = \textstyle\sum_{i,j,k} B_{ijk} C_{ijk} \quad A_{ij} = \textstyle\sum_{k,l} B_{ikl} C_{kj} D_{lj}$$

$$A = B + C \quad a = B^T c + d \quad a = b + c \quad A = \alpha A + \beta B$$

$$A_{ik} = \sum_j B_{ijk} * c_j \quad A = 0 \quad A = B + C + D \quad A = (B + C)(d + e)$$

$$A = \alpha B \quad A = (B + C)d \quad A = BC \quad A_{ilk} = \sum_j B_{ijk} * C_{lj}$$

$$A_{ijl} = \sum_k B_{ijk} * C_{lk} \quad a = BCd \quad a = Bc + b \quad A = A + \alpha I$$

$$A = \alpha B + A \quad A_{jl} = \sum_{i,k} B_{ijk} * C_{il} * D_{kl} \quad a = \alpha Bc + \beta a$$

The Tensor Algebra Compiler is fast and general

$$y = \alpha A^T x + \beta z \quad \boxed{a = Bc}$$

$$a = B^T c \quad A = B + C + D \quad y = b - Ax$$

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Sparse Matrix-Vector Multiplication (SpMV)

The Tensor Algebra Compiler is fast and general

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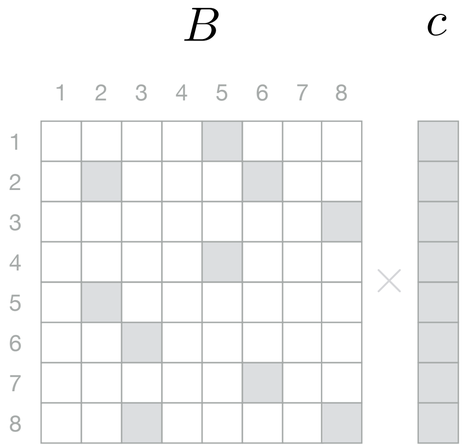
Sparse Matrix-Vector Multiplication (SpMV)



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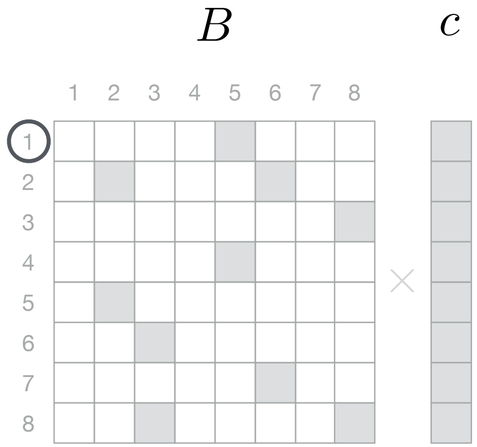
Sparse Matrix-Vector Multiplication (SpMV)



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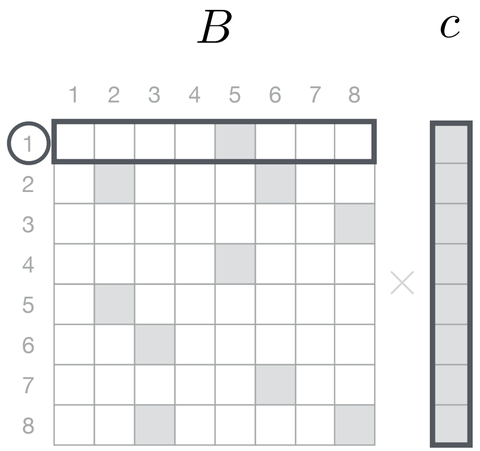
Sparse Matrix-Vector Multiplication (SpMV)



SpMV is competitive with hand-optimized implementations

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$$A = B \odot C \quad A_{ijk} = B_{ijk} + C_{ijk} \quad A_{ij} \sum_k B_{ijk} c_k$$

$$A = B^T \quad \alpha = \sum_{i,j,k} B_{ijk} C_{ijk} \quad A_{ij} = \sum_{k,l} B_{ikl} C_{kj} D_{lj}$$

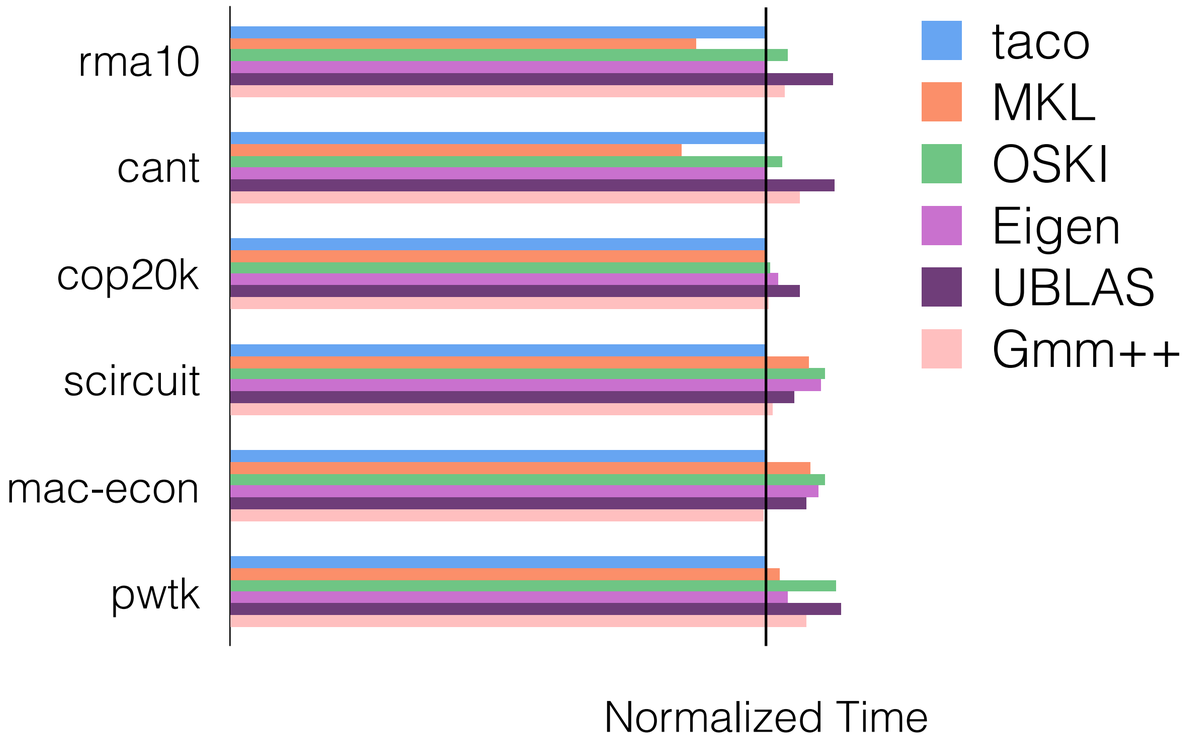
$$A = B + C \quad a = B^T c + d \quad a = b + c \quad A = \alpha A + \beta B$$

$$A_{ik} = \sum_j B_{ijk} * c_j \quad A = 0 \quad A = B + C + D \quad A = (B + C)(d + e)$$

$$A = \alpha B \quad A = (B + C)d \quad A = BC \quad A_{ilk} = \sum_j B_{ijk} * C_{lj}$$

$$A_{ijl} = \sum_k B_{ijk} * C_{lk} \quad a = BCd \quad a = Bc + b \quad A = A + \alpha I$$

$$A = \alpha B + A \quad A_{jl} = \sum_{i,k} B_{ijk} * C_{il} * D_{kl} \quad a = \alpha Bc + \beta a$$



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$$A = \alpha B \quad A = (B + C)d \quad A = BC \quad A_{ilk} = \sum_j B_{ijk} * C_{lj}$$

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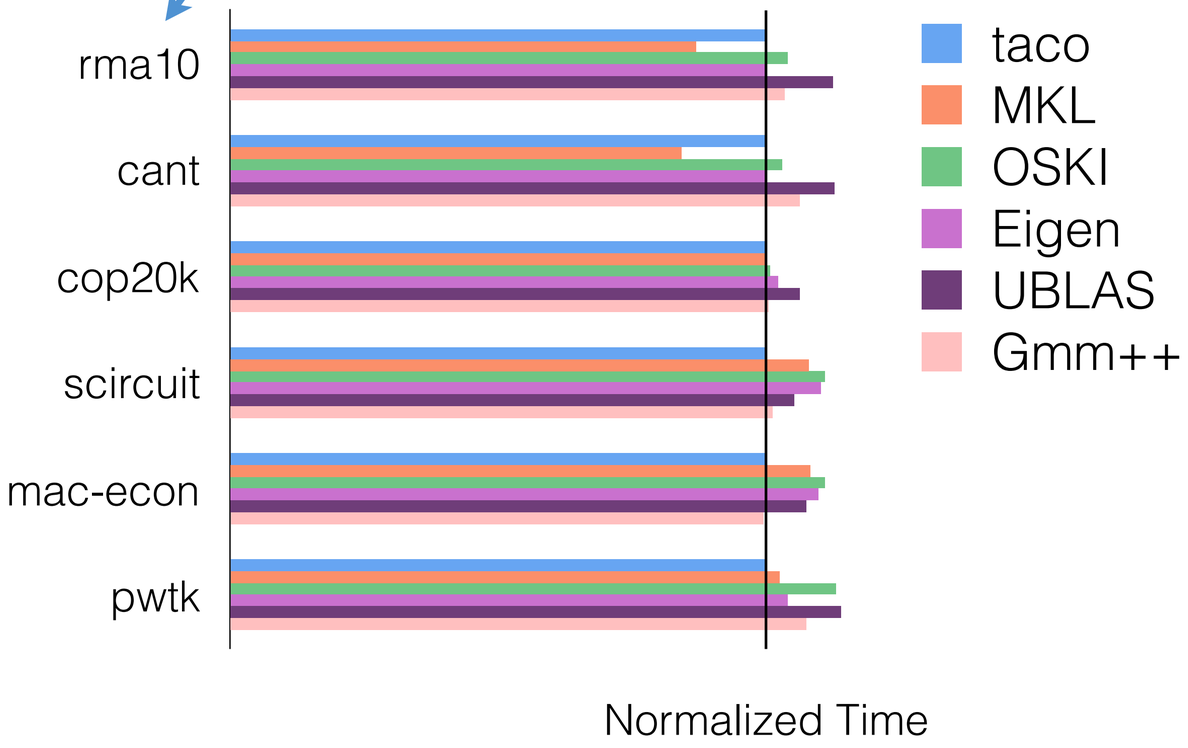
Florida Sparse Matrix Collection



SpMV is competitive with hand-optimized implementations

$$y = \alpha A^T x + \beta z \quad \boxed{a = Bc}$$

$$a = B^T c \quad A = B + C + D \quad y = b - Ax$$

$$A = B \quad a = Bc + a \quad A_{ijk} = \sum_l B_{ijl} C_{kl} \quad A = B \circ (CD)$$

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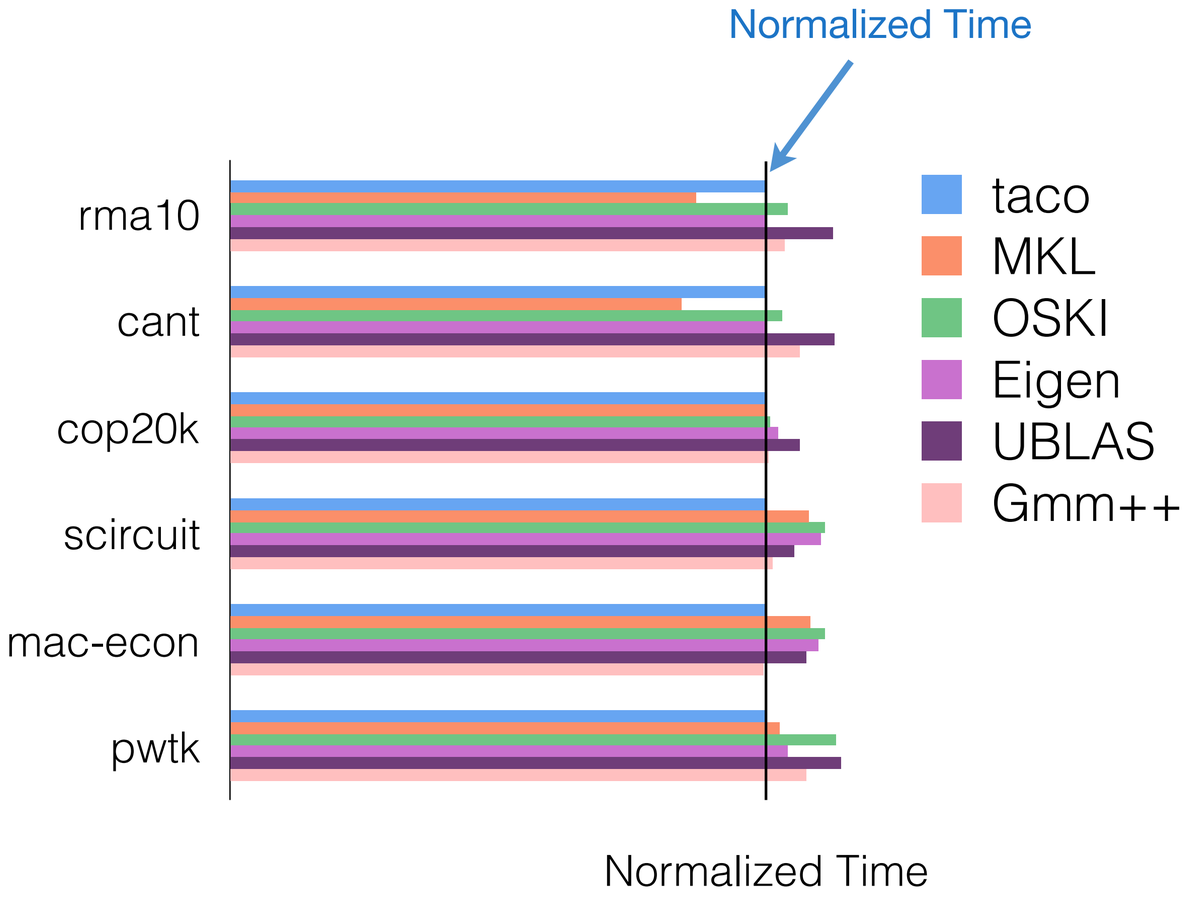
$$A = \alpha B + A \quad A_{jl} = \sum_{i,k} B_{ijk} * C_{il} * D_{kl} \quad a = \alpha Bc + \beta a$$



SpMV is parallel

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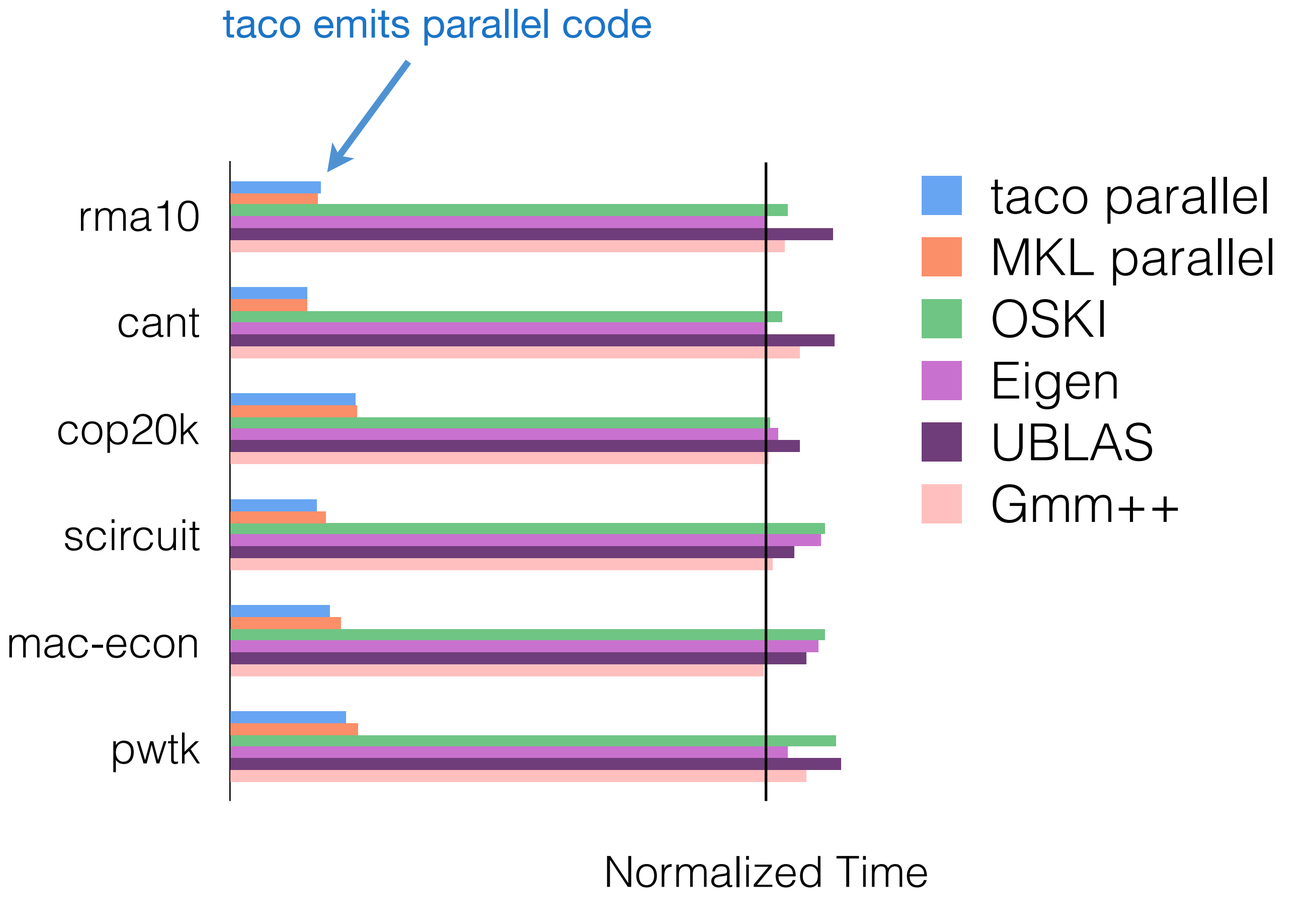

SDDMM must be computed in a single kernel

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Sampled Dense-Dense Matrix Multiplication (SDDMM)

SDDMM must be computed in a single kernel

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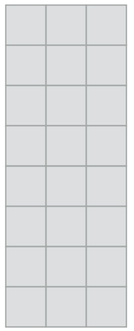
Sampled Dense-Dense Matrix Multiplication (SDDMM)

C



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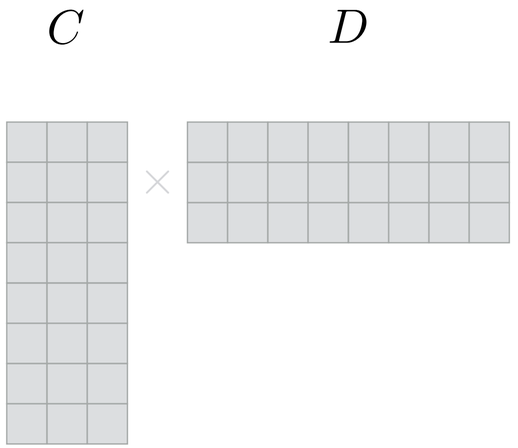

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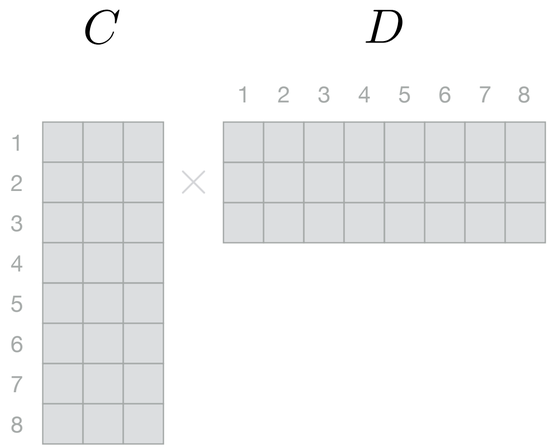

64 inner product

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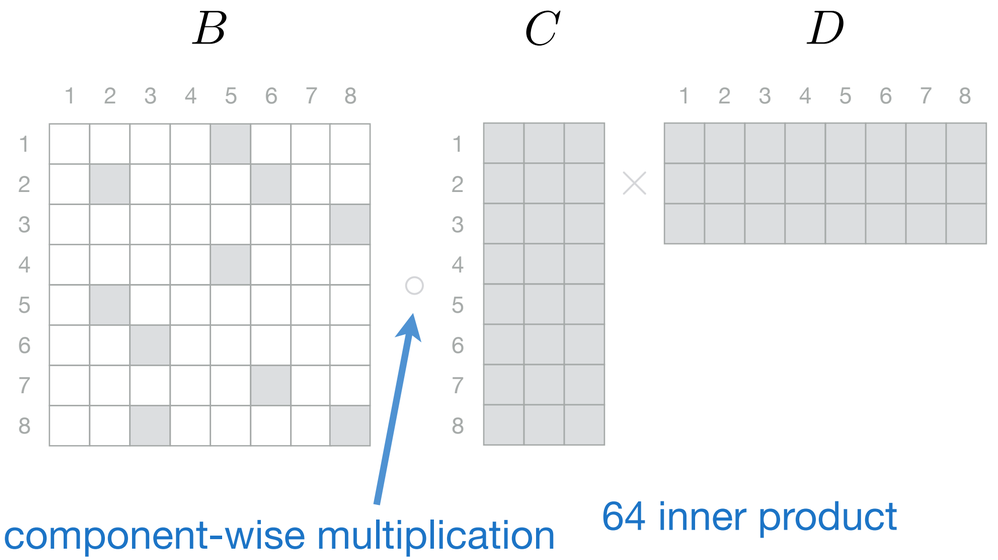

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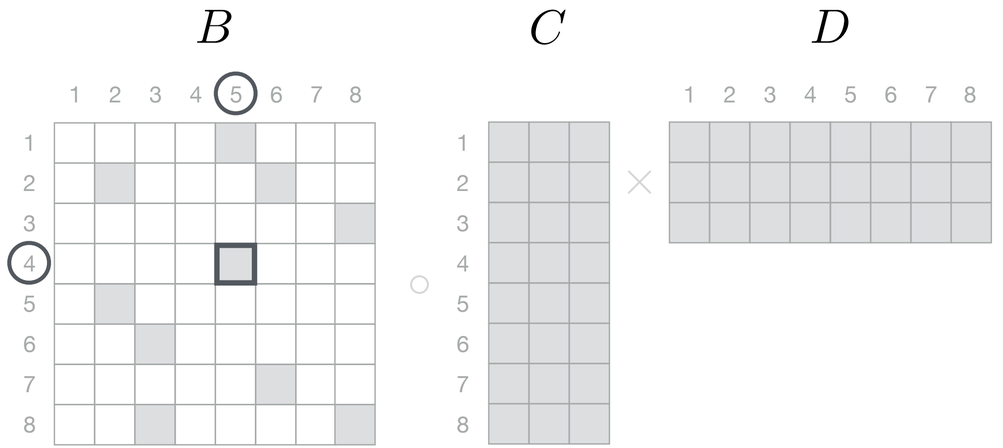

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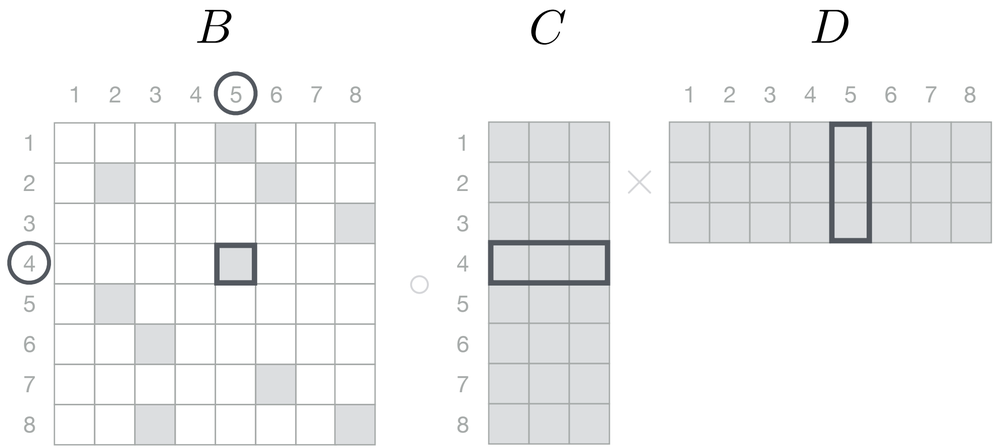
Sampled Dense-Dense Matrix Multiplication (SDDMM)



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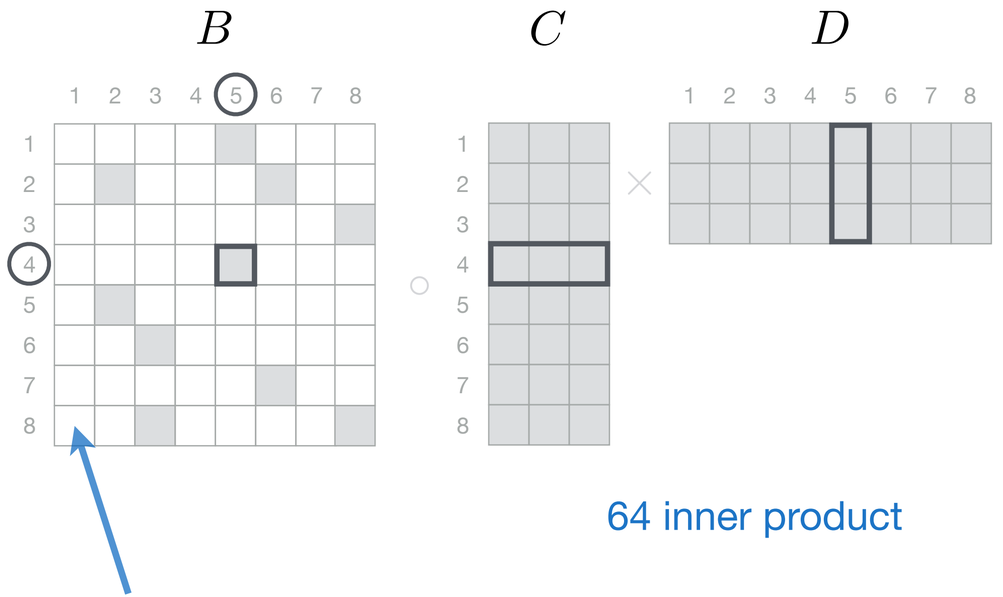
Sampled Dense-Dense Matrix Multiplication (SDDMM)



64 inner product

this dot product need not be computed

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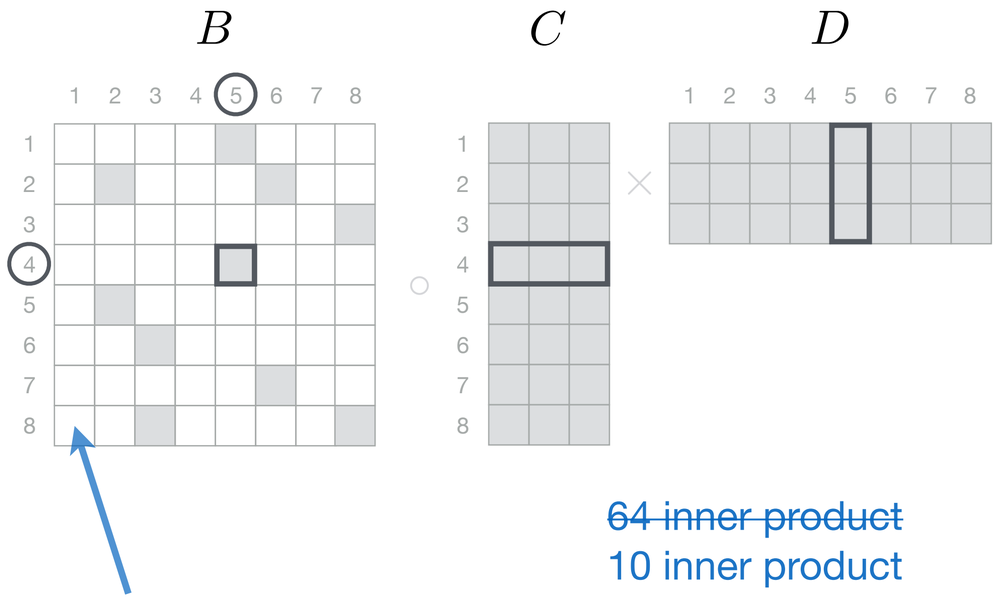

this dot product need not be computed

~~64 inner product~~

10 inner product

SDDMM must be computed in a single kernel

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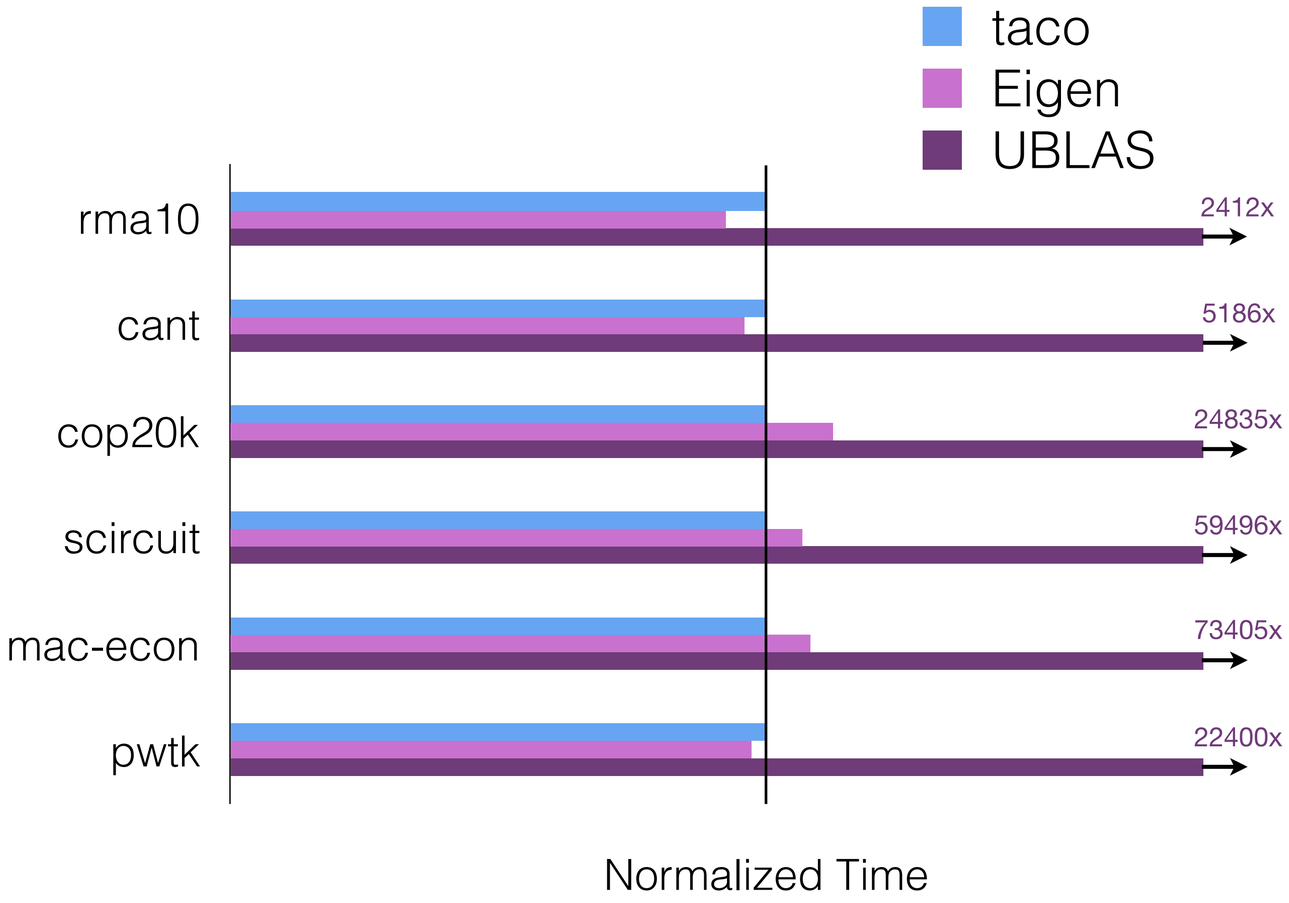

MTTKRP must be computed in a single kernel

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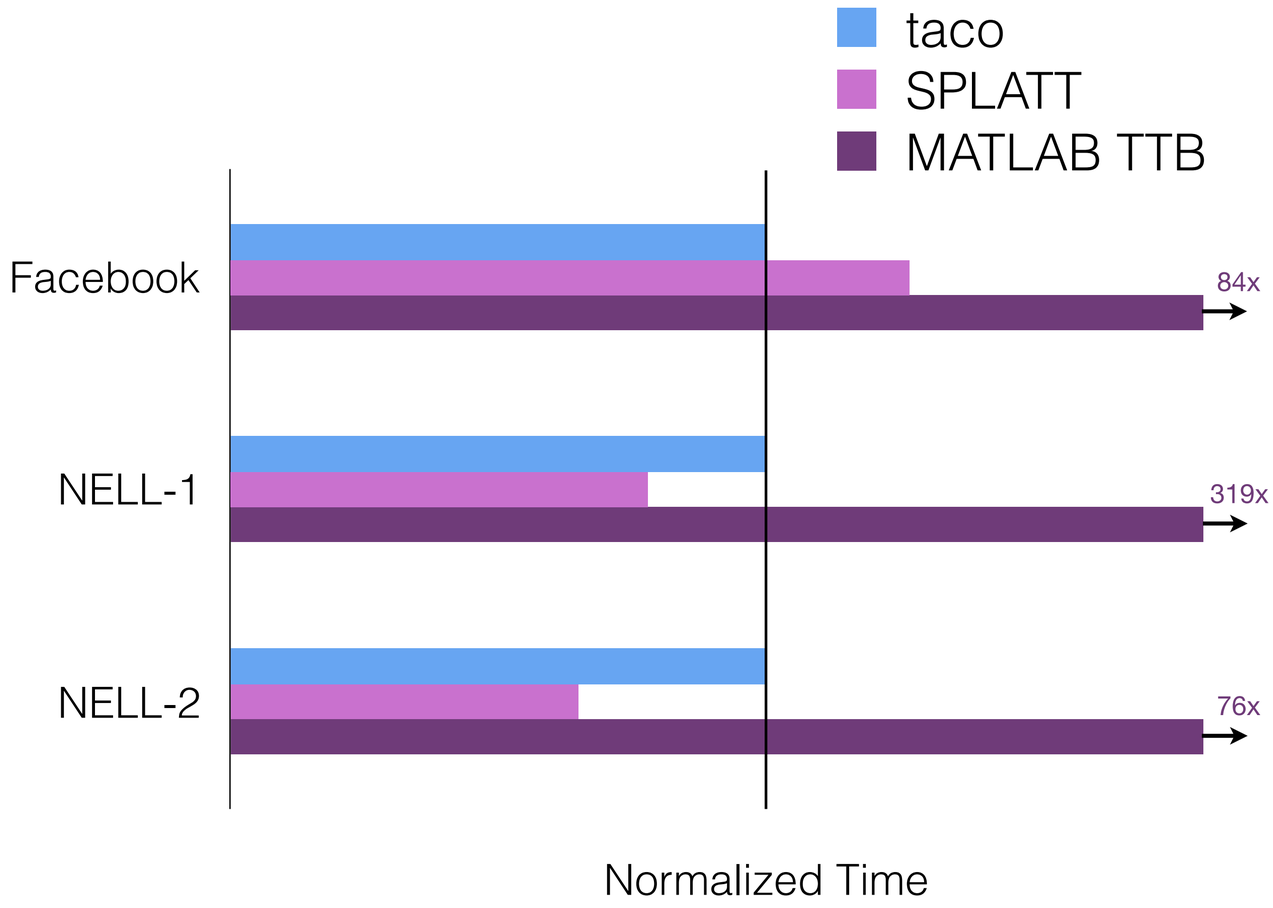
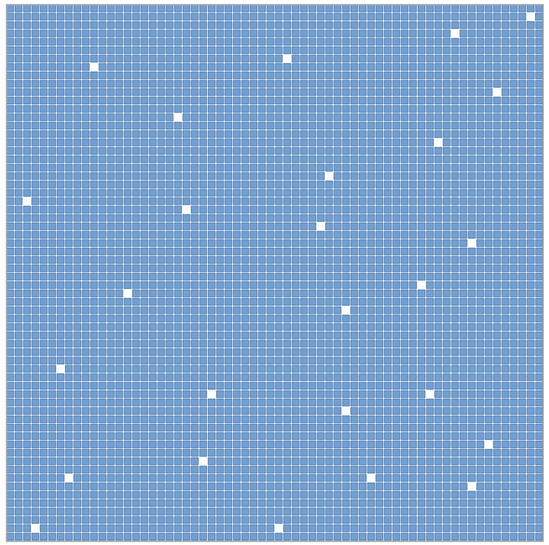
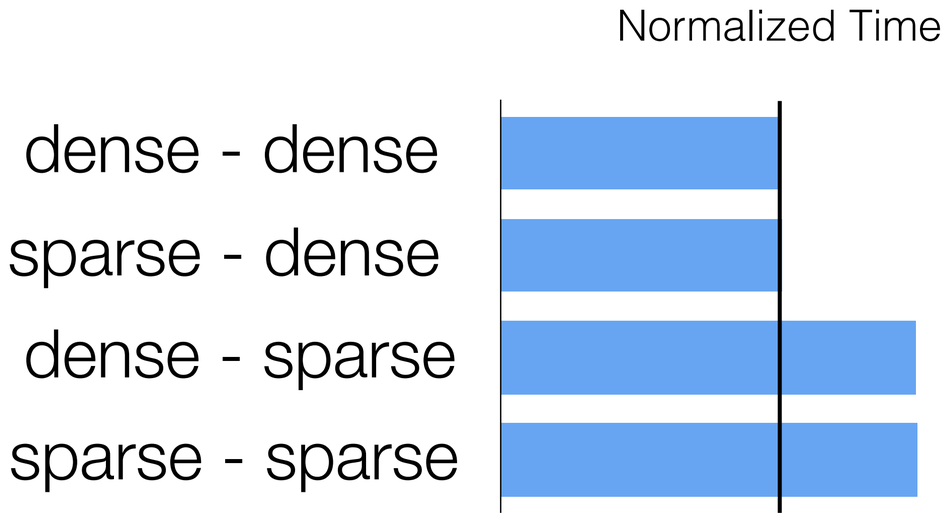


One size does not fit all - different matrices need different formats

Dense Matrix





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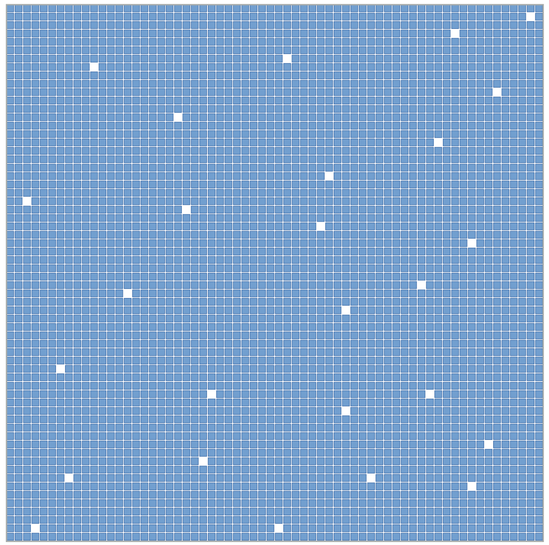

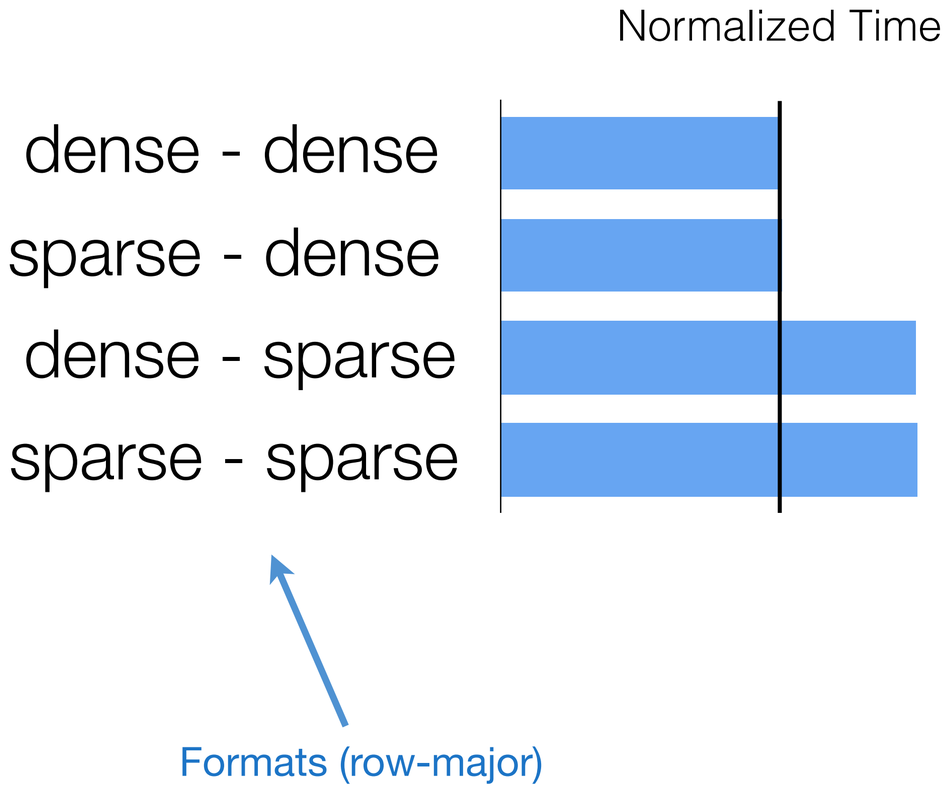

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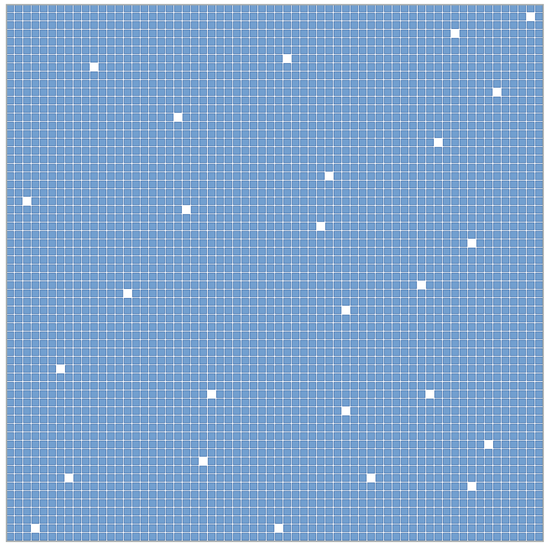


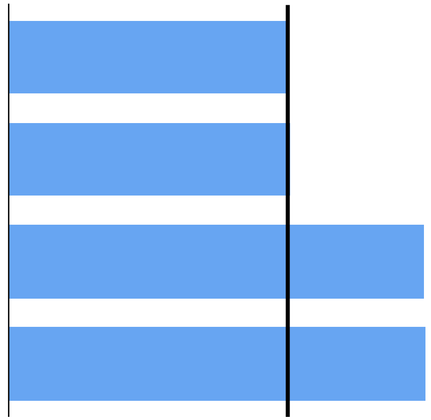
Normalized Time

dense - dense

sparse - dense

dense - sparse

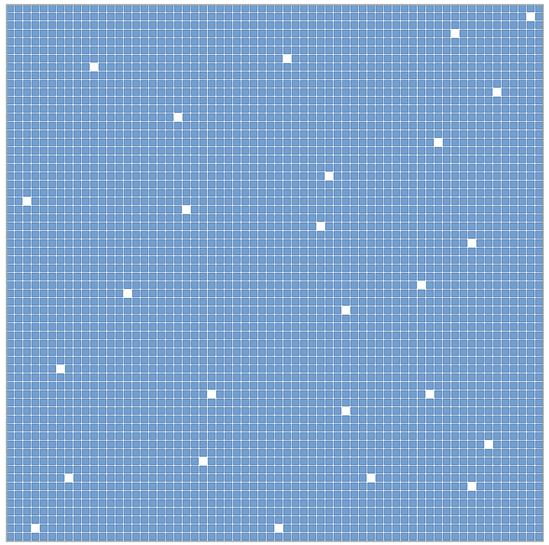
sparse - sparse



$$a_i = \sum_j B_{ij}\, c_j$$

One size does not fit all - different matrices need different formats

Dense Matrix



Best format

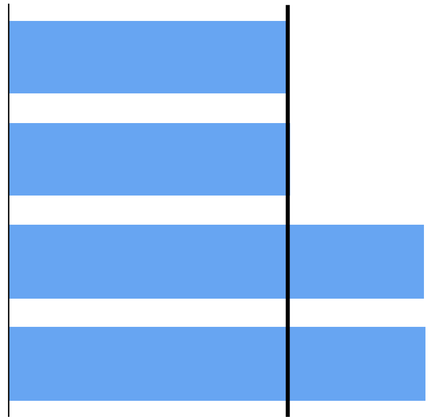
Normalized Time

dense - dense

sparse - dense

dense - sparse

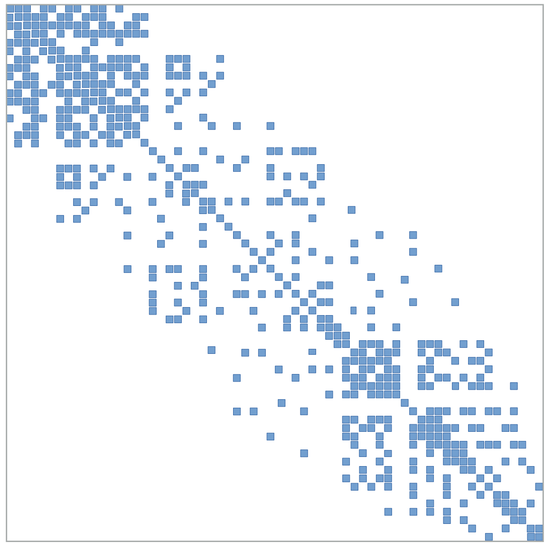
sparse - sparse



$$a_i = \sum_j B_{ij}\, c_j$$

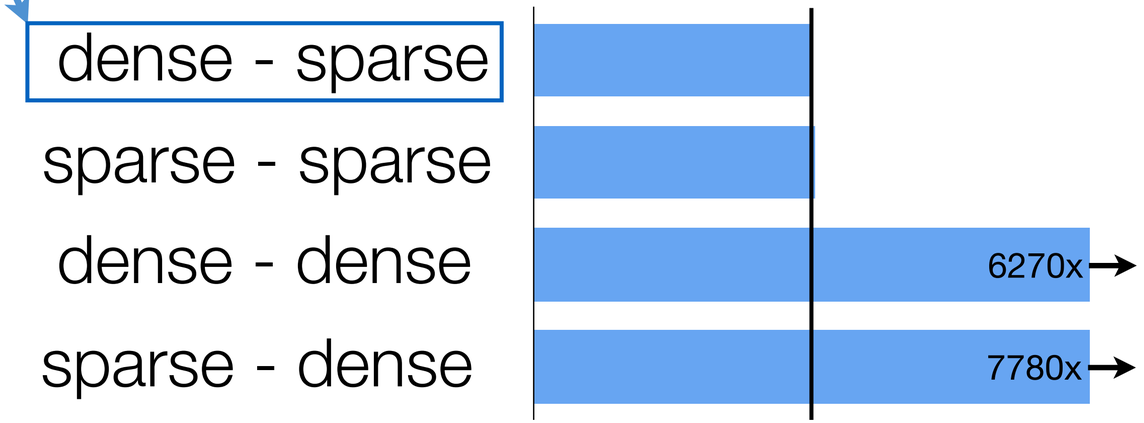
One size does not fit all - different matrices need different formats

Thermal Matrix



Best format

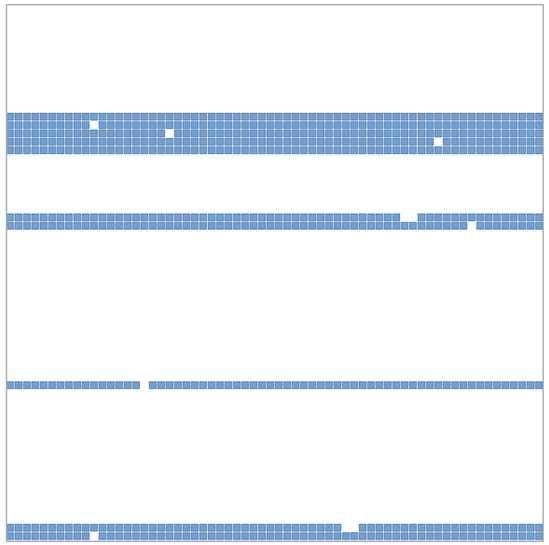
Normalized Time



$$a_i = \sum_j B_{ij}\, c_j$$

One size does not fit all - different matrices need different formats

Row-sliced Matrix



Best format

sparse - dense

sparse - sparse

dense - sparse

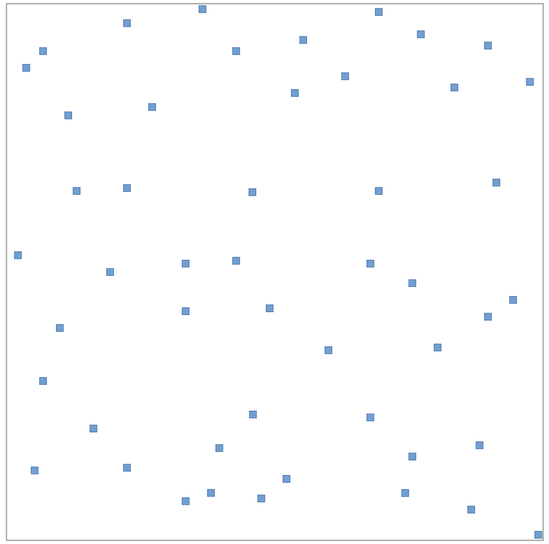
dense - dense

Normalized Time

$$a_i = \sum_j B_{ij}\, c_j$$

One size does not fit all - different matrices need different formats

Hypersparse Matrix



Best format

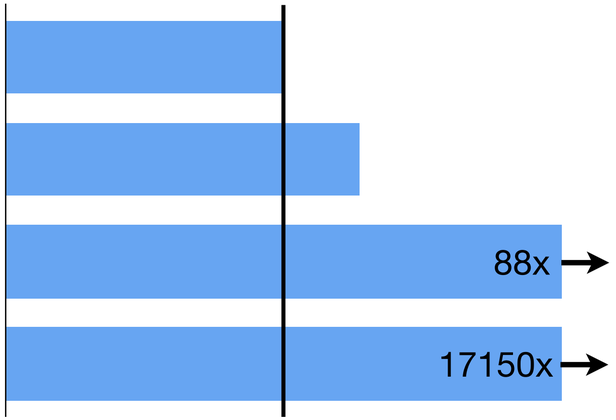
sparse - sparse

dense - sparse

sparse - dense

dense - dense

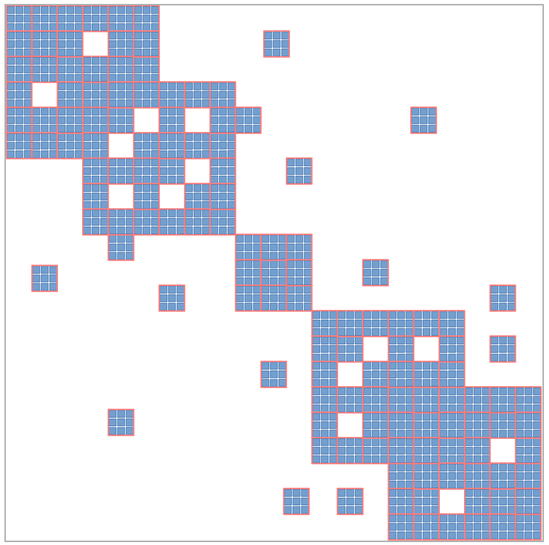
Normalized Time



$$a_i = \sum_j B_{ij}\, c_j$$

Blocked matrices are 4-tensors

Blocked FEM Matrix



3x3 blocks

dense - sparse - dense - dense

sparse - sparse - dense - dense

dense - sparse

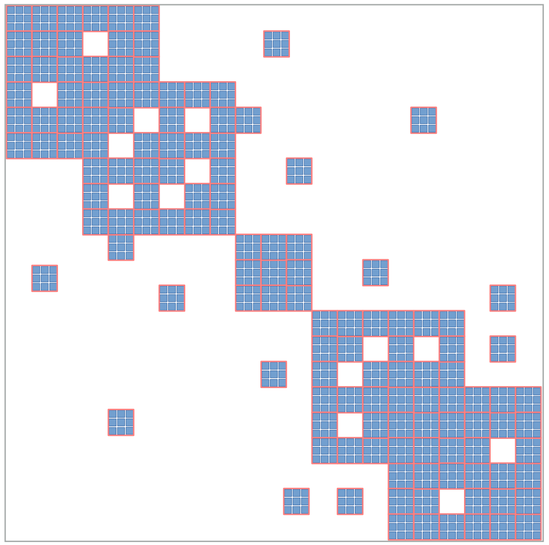
sparse - sparse

$$a_i = \sum_j B_{ij}\, c_j$$

Normalized Time

Blocked matrices are 4-tensors

Blocked FEM Matrix



4-tensor

dense - sparse - dense - dense

sparse - sparse - dense - dense

dense - sparse

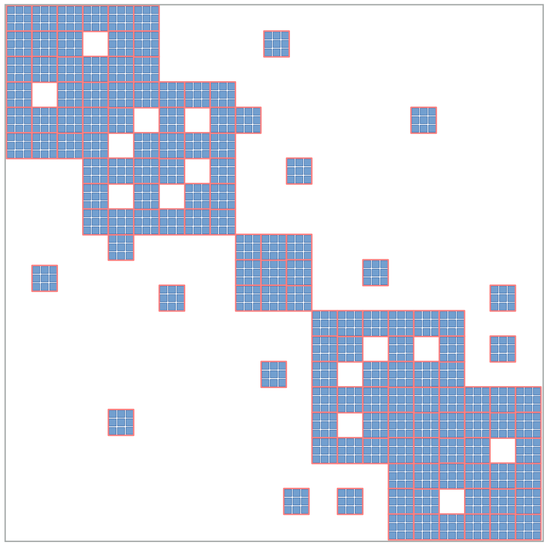
sparse - sparse

Normalized Time

$$a_i = \sum_j B_{ij}\, c_j$$

Blocked matrices are 4-tensors

Blocked FEM Matrix



$$a_{ik} = \sum_j \sum_l B_{ijkl}\, c_{jl}$$

dense - sparse - dense - dense

sparse - sparse - dense - dense

dense - sparse

sparse - sparse

$$a_i = \sum_j B_{ij}\, c_j$$

Normalized Time

Future Work (some of it)

Portability/Acceleration

Formats

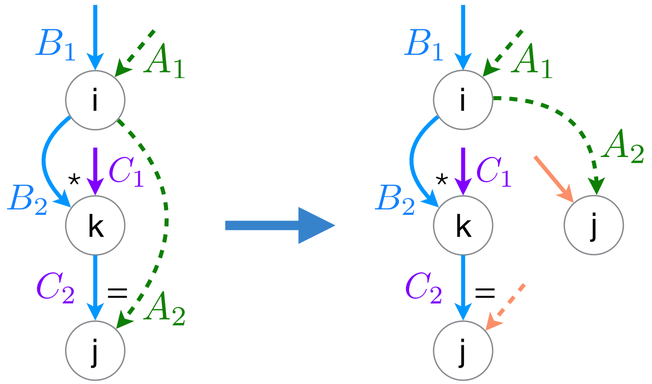
COO

DIA

ELL

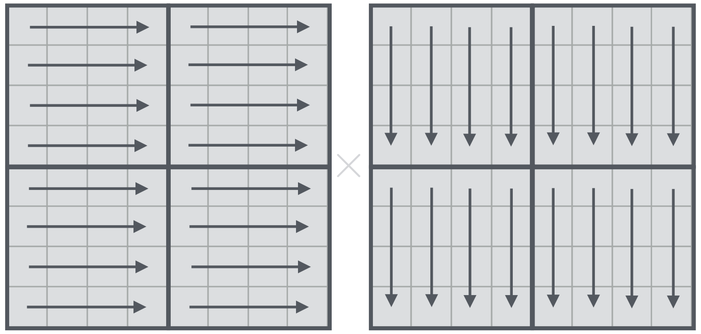
Hash maps

Workspaces



$$A_{ij} = B_{ik}C_{kj}$$

Scheduling Language



Semirings

```
(+,*)          float
(max,+)        double
(min,max)      complex
(∧,∨)          int
(f(),g())      bool
```

Policy

Inspector-Executor

ML

Sampling

Heuristics

Autotuning

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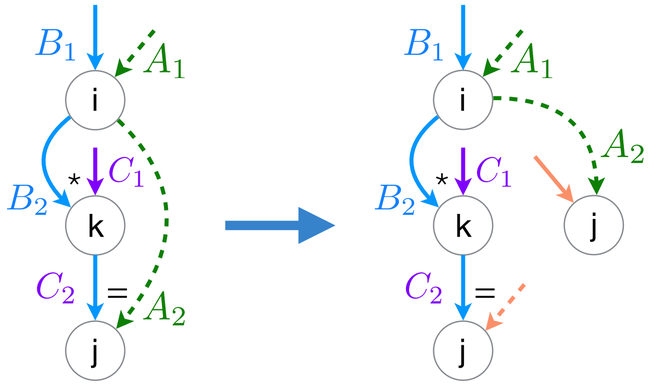

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Policy

Inspector-Executor

ML

Sampling

Heuristics

Autotuning

Tensor Algebra Compiler Library and Tools

C++ Tensor Library

```
Format  CSR({Dense,Sparse});
Format  CSF({Sparse,Sparse,Sparse});
Format SVEC({Sparse});

Tensor<double> A({1024,1024}, CSR);
Tensor<double> B = read("B.tns", CSF);
Tensor<double> c = read("c.tns", SVEC);

Var i, j, k;
A(i,j) = B(i,j,k) * c(k);

A.evaluate();  // Compiles, assemble, and compute
               // the expression
```

tensor-compiler.org

github.com/tensor-compiler/taco

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Online Code Generator

tensor-compiler.org/codegen

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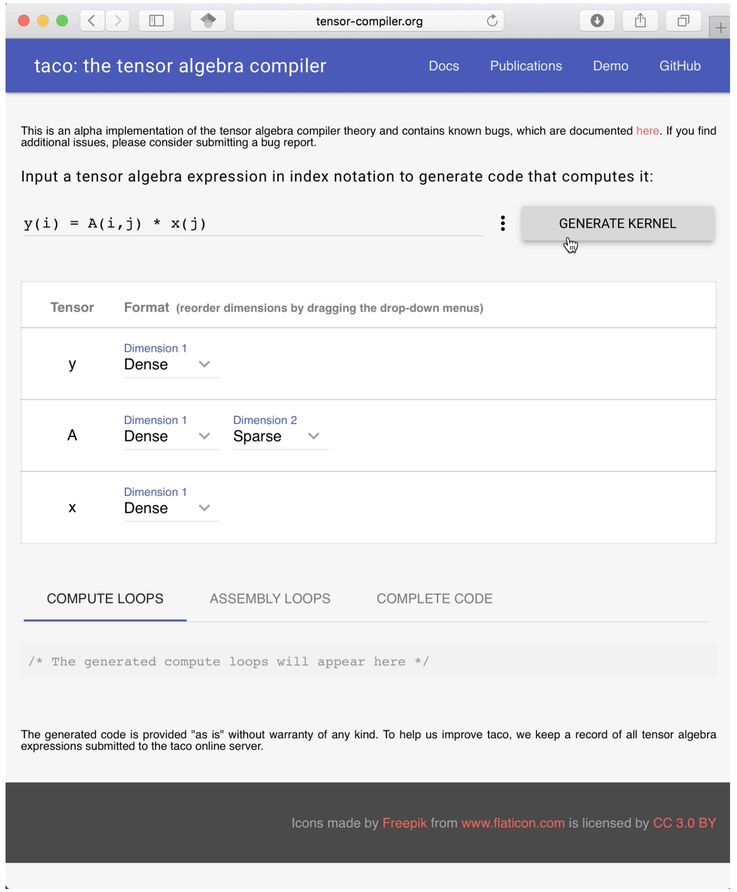

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